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Comments on Low-Energy Two-Photon Processes at LEP

—Version II—

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abstract

A reasonably self-contained account is given of the systematics of two-photon processes at low energy, i.e. $e^+ e^- \to e^+ e^- X$ with $\gamma\gamma \to X$, where X is a low-mass system of hadrons produced in $e^+ e^-$ collisions at LEP.

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1 Introduction

This report represents an attempt to give a reasonably self-contained account of the systematics of two-photon processes at low energy, i.e.

$$e^+ e^- \to e^+ e^- X, \qquad \gamma\gamma \to X$$

where X is a low-mass system of hadrons produced in $e^+ e^-$ collisions at LEP.

The helicity formalism of Jacob and Wick[1][2] has been used, in Section 2, to describe two-photon systems of varying virtuality. Relativistic kinematics and invariant phase-space elements are given in Section 3. This section represents an extension of a work done in the context of a RHIC proposal[3][4]. Differential cross sections appropriate for the reaction of interest are dealt with in Section 4. Description of s-channel resonances in two-photon processes is given in Section 5.

The reader may wish to consult an informal report[5] for a discussion of exotic mesons at LEP. It also contains a simplified treatment of the materials covered in Sections 3 and 4. Section 5 of this report is in fact a reproduction of Appendix B of the informal report[5].

Many of the work cited in this preprint, including the current one as well as several other relevant work, could be found at a website

$$\mathbf{http://cern.ch/suchung}$$

which may be accessed without a password.

2 Two-Photon Fusion Processes

The idea is to use the LEP detectors to study the s-channel formation of J^{PC}-exotic mesons, in the processes

$$\begin{aligned} \gamma(t_1) + \gamma(t_2) &\to n\bar{n} + n\bar{n} \\ \gamma(t_1) + \gamma(t_2) &\to c\bar{c} + n\bar{n} \end{aligned} \tag{1}$$

where $\gamma(t)$ stands for the photon with a momentum transfer t. Let p_a and p_b be the lab momenta for the initial e^+ and e^-, respectively, and let p'_a and p'_b be those in the final state. Then, the momentum transfers are given by

$$\begin{aligned} t_1 &= (\, p_a - p'_a\,)^2 \\ t_2 &= (\, p_b - p'_b\,)^2 \end{aligned} \tag{2}$$

Assume for the purpose of discussion that $|t_1| < |t_2|$. For analysis of the states produced in the s-channel, the z-axis may be chosen such that

$$\vec{z} \;\propto\; (\,\vec{p}_a - \vec{p}_a\,'\,) - (\,\vec{p}_b - \vec{p}_b\,'\,) \tag{3}$$

Of course, if $|t_1| > |t_2|$, then the subscripts a and b should be interchanged. It is clear that with this choice of the axis one should not apply Bose symmetrization to the system containing two photons $\gamma(t_1)$ and $\gamma(t_2)$, for they are distinguishable by their momentum transfer.

A system of two identical particles, e.g. two off-shell photons at $t_1 = t_2$, must have[2]

$$L + S = \text{even} \tag{4}$$

where L is the orbital angular momentum in the rest frame and S is the total intrinsic spin, i.e. $S = 0$, 1 or 2. For an exotic meson under investigation, one must have $L = S = 1$ for $J^{PC} = 1^{-+}$, and $L = 3$ and $S = 1$ for $J^{PC} = 3^{-+}$. So, if Bose symmetrization applies, then exotic mesons are not possible for $S = 0$ or 2. To the extent that two photons $\gamma(t_1)$ and $\gamma(t_2)$ are distinguishable by their momentum transfer, all possible L and S are allowed. In particular, the $J^{PC} = 1^{-+}$ ($J^{PC} = 3^{-+}$) exotic meson is possible with $L = 1$ and $S = 0$ ($S = 2$).

Special relationships exist if a state decays to two identical massless photons. The decay amplitude may be written[2][1], in terms of the helicity-coupling amplitudes $F^a_{\lambda_1\,\lambda_2}$,

$$A \;\propto\; F^a_{\lambda_1\,\lambda_2}\, D^{J\,*}_{\Lambda\,\Lambda}(0,0,0) \tag{5}$$

where $\Lambda = \lambda_1 - \lambda_2$ and $a = J^P$. Note that $D^J_{\Lambda\,\Lambda}(0,0,0) = 1$ only if $J \geq |\Lambda|$. We have chosen a coordinate system in which the direction given by the two photons in their center-of-mass system is the z-axis and the azimuthal angle ϕ is fixed at zero. Note that the D^J must come with identical M components for it to be non-zero. Because of Bose symmetrization, one has[2][1]

$$F^a_{\lambda_1\,\lambda_2} = (-)^J\, F^a_{\lambda_2\,\lambda_1} \tag{6}$$

Since the decay particles are massless, the helicities cannot be zero. So, if J = odd, then $F^a_{\pm\pm} = 0$ and the only allowed helicities are $\{\lambda_1, \lambda_2\} = \{\pm, \mp\}$ or $\Lambda = \pm 2$. But then the D-function is zero if $J = 1$, and one concludes that a spin-1 state cannot couple to two photons. There exists an additional constraint on the helicity-coupling amplitudes, coming from the parity conservation in the decay, i.e.

$$F^a_{\lambda_1\,\lambda_2} = \nu_1\,\nu_2\,\nu_a\; F^a_{-\lambda_1\,-\lambda_2} \tag{7}$$

where ν is the naturality of the particle, given by $\nu_1 = \eta_1(-)^{J_1}$, $\nu_2 = \eta_2(-)^{J_2}$ and $\nu_a = \eta_a(-)^J$. For photons one must have $\nu_1 = \nu_2 = +1$. If J = odd, $\{\lambda_1, \lambda_2\} = \{\pm, \mp\}$ from (6), and (7) can be satisfied only if $\nu_a = (-)^J$ or $\eta_a = +1$. This means that no exotic states, for any J, can couple to on-shell $\gamma\gamma$.

The reason for a scrupulous choice of the coordinate system in the previous paragraph has been to avoid this problem: to repeat, a spin-1 state can couple to two photons, as long as they can be distinguished by their momentum transfer.

Now consider a case in which one or both photons are off-shell but neither is tagged. Then one has no way of distinguishing the two photons, and so the Bose symmetrization applies once again. The allowed helicities and spin-parities, derived from (6) and (7), are tabulated in Table Ia.

Table Ia: Allowed J^{PC}'s in two-photon Proceses
(Bose symmetrization imposed)

$\{\lambda_1, \lambda_2\}$	Λ	J^{PC}
$\{\pm, \pm\}$	0	even$^{\pm+}$, $J \geq 0$
$\{\pm, \mp\}$	± 2	J^{++}, $J \geq 2$
$\{0,\ 0\}$	0	even^{++}, $J \geq 0$
$\{0,\ \mp\}$	± 1	$J^{\pm+}$, $J \geq 1$
$\{\pm,\ 0\}$	± 1	$J^{\pm+}$, $J \geq 1$

A comment on the interference effects between different states listed above is in order. For the purpose, it is best to start with the distribution function $I(\tau)$, where $g(\tau)\,\mathrm{d}\tau$ is the relativistic phase-space element. It may be expressed, in terms of the helicity-coupling amplitudes of (5),

$$I(\tau) = \sum_{\lambda_1,\lambda_2} \Big| \sum_{a} F^{a}_{\lambda_1\,\lambda_2}(\tau) \Big|^2 = \sum_{a\,a'} I_{a\,a'}(\tau) \tag{8}$$

where

$$I_{a\,a'}(\tau) = \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_1\,\lambda_2}(\tau)\, F^{a'\,*}_{\lambda_1\,\lambda_2}(\tau) \tag{9}$$

From (6), one sees that

$$\begin{aligned} I_{a\,a'}(\tau) &= \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_2\,\lambda_1}(\tau)\, F^{a'\,*}_{\lambda_2\,\lambda_1}(\tau) \\ &= (-)^{J-J'} \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_1\,\lambda_2}(\tau)\, F^{a'\,*}_{\lambda_1\,\lambda_2}(\tau) \\ &= (-)^{J-J'}\, I_{a\,a'}(\tau) \end{aligned} \tag{10}$$

while, from (7), one finds

$$\begin{aligned} I_{a\,a'}(\tau) &= \sum_{\lambda_1,\lambda_2} F^{a}_{-\lambda_1\,-\lambda_2}(\tau)\, F^{a'\,*}_{-\lambda_1\,-\lambda_2}(\tau) \\ &= \nu_a\,\nu_{a'} \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_1\,\lambda_2}(\tau)\, F^{a'\,*}_{\lambda_1\,\lambda_2}(\tau) \\ &= \nu_a\,\nu_{a'}\, I_{a\,a'}(\tau) \end{aligned} \tag{11}$$

So one concludes that the interference term should be zero, i.e. $I_{a\,a'} = 0\,(a \neq a')$, unless

$$J - J' = \text{even} \qquad \underline{\text{and}} \qquad \nu_a = \nu_{a'} \tag{12}$$

It is clear, in addition, that two states with different $\{\lambda_1, \lambda_2\}$ cannot interfere. There are in general nine elements in the set $\{\lambda_1, \lambda_2\}$, but because of the symmetry (6) and (7), there are only four independent elements; $\{+,+\}\,(\Lambda = 0)$, $\{+,-\}(\Lambda = 2)$, $\{0,0\}(\Lambda = 0)$ and $\{+,0\}(\Lambda = 1)$. There cannot be interference terms among the four elements listed above. So the partial-wave amplitudes $A_{\lambda_1\,\lambda_2}(J^{PC}\,\Lambda;\tau)$ are grouped together into new amplitudes $B^{(k\;\nu)}_{\lambda_1\,\lambda_2}(\Lambda;\tau)$, where the superscript k stands for either e (J=even) or o (J=odd) and ν denotes the naturality. Grouped into different Λ's, they are, for $\Lambda = 0$,

$$\begin{aligned} B^{(e+)}_{++}(0;\tau) &= A_{++}(0^{++}\,0;\tau) + A_{++}(2^{++}\,0;\tau) + A_{++}(4^{++}\,0;\tau) + \cdots \\ B^{(e-)}_{++}(0;\tau) &= A_{++}(0^{-+}\,0;\tau) + A_{++}(2^{-+}\,0;\tau) + A_{++}(4^{-+}\,0;\tau) + \cdots \\ B^{(e+)}_{0\,0}(0;\tau) &= A_{0\,0}(0^{++}\,0;\tau) + A_{0\,0}(2^{++}\,0;\tau) + A_{0\,0}(4^{++}\,0;\tau) + \cdots \end{aligned} \tag{13}$$

for $\Lambda = 1$,

$$
\begin{aligned}
B^{(e+)}_{+0}(1;\tau) &= A_{+0}(2^{++}\,1;\tau) + A_{+0}(4^{++}\,1;\tau) + A_{+0}(6^{++}\,1;\tau) + \cdots \\
B^{(e-)}_{+0}(1;\tau) &= A_{+0}(2^{-+}\,1;\tau) + A_{+0}(4^{-+}\,1;\tau) + A_{+0}(6^{-+}\,1;\tau) + \cdots \\
B^{(o-)}_{+0}(1;\tau) &= A_{+0}(1^{++}\,1;\tau) + A_{+0}(3^{++}\,1;\tau) + A_{+0}(5^{++}\,1;\tau) + \cdots \\
B^{(o+)}_{+0}(1;\tau) &= A_{+0}(1^{-+}\,1;\tau) + A_{+0}(3^{-+}\,1;\tau) + A_{+0}(5^{-+}\,1;\tau) + \cdots
\end{aligned}
\tag{14}
$$

and, for $\Lambda = 2$,

$$
\begin{aligned}
B^{(e+)}_{+-}(2;\tau) &= A_{+-}(2^{++}\,2;\tau) + A_{+-}(4^{++}\,2;\tau) + A_{+-}(6^{++}\,2;\tau) + \cdots \\
B^{(o-)}_{+-}(2;\tau) &= A_{+-}(3^{++}\,2;\tau) + A_{+-}(5^{++}\,2;\tau) + A_{+-}(7^{++}\,2;\tau) + \cdots
\end{aligned}
\tag{15}
$$

Note that the states with $J^{PC} = 1^{-+}$, $J^{PC} = 3^{-+}$ and $J^{PC} = 5^{-+}$ are exotic and hence represent the mesons which cannot be quarkonia.

The overall distribution function is now given by a sum of the absolute squares of 9 non-interterfering amplitudes, as follows:

$$
\begin{aligned}
I(\tau) = {} & \left|B^{(e+)}_{++}(0;\tau)\right|^2 + \left|B^{(e-)}_{++}(0;\tau)\right|^2 + \left|B^{(e+)}_{0\,0}(0;\tau)\right|^2 \\
& + \left|B^{(e+)}_{+0}(1;\tau)\right|^2 + \left|B^{(e-)}_{+0}(1;\tau)\right|^2 + \left|B^{(o+)}_{+0}(1;\tau)\right|^2 + \left|B^{(o-)}_{+0}(1;\tau)\right|^2 \\
& + \left|B^{(e+)}_{+-}(2;\tau)\right|^2 + \left|B^{(o-)}_{+-}(2;\tau)\right|^2
\end{aligned}
\tag{16}
$$

If one assumes that both photons are on-shell (which may be a good approximation), then the distribution function is a sum of four terms only; the first two plus the last two terms in (16). Assume further that for a limited mass range, e.g. $\sqrt{s} < 3.0$ GeV, one can confine the partial waves to those with $J \le 2$. In this case the last term of (16) disappears as well, since it consists of the states $|3^{++}\,2\rangle$ and possibly those with $J \ge 4$ which *do not interfere* with the first three terms. The distribution function can now be written

$$
\begin{aligned}
I(\tau) = {} & \left|A_{++}(0^{++}\,0;\tau) + A_{++}(2^{++}\,0;\tau)\right|^2 \\
& + \left|A_{++}(0^{-+}\,0;\tau) + A_{++}(2^{-+}\,0;\tau)\right|^2 + \left|A_{+-}(2^{++}\,2;\tau)\right|^2
\end{aligned}
\tag{17}
$$

Note the extreme simplicity of this formula; in particular, the partial wave $|2^{++}\,2\rangle$ is produced in isolation, i.e. with no interference with any other waves in the problem. However, note that the two states $|0^{++}\,0\rangle$ and $|2^{++}\,0\rangle$ are produced with 100% coherence between them, as are the two states $|0^{-+}\,0\rangle$ and $|2^{-+}\,0\rangle$.

Consider now a case in which one of the photons is tagged. It is clear that the photons can now be distinguished and so the Bose symmetrization does not apply. But the constraint (7) due to parity conservation must remain valid. The allowed J^{PC}'s for this case are listed in Table Ib.

Table Ib: Allowed J^{PC}'s in two-photon Proceses
(without Bose symmetrization)

$\{\lambda_1, \lambda_2\}$	Λ	J^{PC}
$\{\pm, \pm\}$	0	$J^{\pm+}$, $J \geq 0$
$\{\pm, \mp\}$	± 2	$J^{\pm+}$, $J \geq 2$
$\{0,\ 0\}$	0	even^{++}, odd^{-+}, $J \geq 0$
$\{0,\ \mp\}$	± 1	$J^{\pm+}$, $J \geq 1$
$\{\pm,\ 0\}$	± 1	$J^{\pm+}$, $J \geq 1$

There should be five independent elements in the set $\{\lambda_1, \lambda_2\}$; $\{+,\ +\}$ $(\Lambda = 0)$, $\{+,\ -\}(\Lambda = 2)$, $\{0,\ 0\}(\Lambda = 0)$, $\{0,\ -\}(\Lambda = 1)$ and $\{+,\ 0\}(\Lambda = 1)$. If the untagged photon is very nearly on-shell, e.g. $\lambda_1 = 0$ is absent, then there would be just three independent elements, consisting of the first, second and the last element on the list above.

The partial-wave amplitudes $A_{\lambda_1\,\lambda_2}(J^{PC}\,\Lambda;\tau)$ can be grouped together into new amplitudes $B^{(\nu)}_{\lambda_1\,\lambda_2}(\Lambda;\tau)$, where ν denotes the naturality. Grouped into different Λ's, they are, for $\Lambda = 0$,

$$\begin{aligned}
B^{(+)}_{++}(0;\tau) &= A_{++}(0^{++}\,0;\tau) + A_{++}(1^{-+}\,0;\tau) + A_{++}(2^{++}\,0;\tau) + \cdots \\
B^{(-)}_{++}(0;\tau) &= A_{++}(0^{-+}\,0;\tau) + A_{++}(1^{++}\,0;\tau) + A_{++}(2^{-+}\,0;\tau) + \cdots \\
B^{(+)}_{0\,0}(0;\tau) &= A_{0\,0}(0^{++}\,0;\tau) + A_{0\,0}(1^{-+}\,0;\tau) + A_{0\,0}(2^{++}\,0;\tau) + \cdots
\end{aligned} \tag{18}$$

for $\Lambda = 1$,

$$\begin{aligned}
B^{(+)}_{+\,0}(1;\tau) &= A_{+\,0}(1^{-+}\,1;\tau) + A_{+\,0}(2^{++}\,1;\tau) + A_{+\,0}(3^{-+}\,1;\tau) + \cdots \\
B^{(-)}_{+\,0}(1;\tau) &= A_{+\,0}(1^{++}\,1;\tau) + A_{+\,0}(2^{-+}\,1;\tau) + A_{+\,0}(3^{++}\,1;\tau) + \cdots \\
B^{(+)}_{0-}(1;\tau) &= A_{0-}(1^{-+}\,1;\tau) + A_{0-}(2^{++}\,1;\tau) + A_{0-}(3^{-+}\,1;\tau) + \cdots \\
B^{(-)}_{0-}(1;\tau) &= A_{0-}(1^{++}\,1;\tau) + A_{0-}(2^{-+}\,1;\tau) + A_{0-}(3^{++}\,1;\tau) + \cdots
\end{aligned} \tag{19}$$

and, for $\Lambda = 2$,

$$\begin{aligned} B^{(+)}_{+-}(2;\tau) &= A_{+-}(2^{++}\,2;\tau) + A_{+-}(3^{-+}\,2;\tau) + A_{+-}(4^{++}\,2;\tau) + \cdots \\ B^{(-)}_{+-}(2;\tau) &= A_{+-}(2^{-+}\,2;\tau) + A_{+-}(3^{++}\,2;\tau) + A_{+-}(4^{-+}\,2;\tau) + \cdots \end{aligned} \tag{20}$$

The overall distribution function is now given by a sum of the absolute squares of 9 non-interterfering amplitudes, as follows:

$$\begin{aligned} I(\tau) = {} & \left|B^{(+)}_{++}(0;\tau)\right|^2 + \left|B^{(-)}_{++}(0;\tau)\right|^2 + \left|B^{(+)}_{0\,0}(0;\tau)\right|^2 \\ & + \left|B^{(+)}_{+0}(1;\tau)\right|^2 + \left|B^{(-)}_{+0}(1;\tau)\right|^2 + \left|B^{(+)}_{0-}(1;\tau)\right|^2 + \left|B^{(-)}_{0-}(1;\tau)\right|^2 \\ & + \left|B^{(+)}_{+-}(2;\tau)\right|^2 + \left|B^{(-)}_{+-}(2;\tau)\right|^2 \end{aligned} \tag{21}$$

If the untagged photon is very nearly on-shell, i.e. $\lambda_1 = 0$ is absent, then the distribution function consists of 6 non-interterfering amplitudes, given by

$$\begin{aligned} I(\tau) = {} & \left|B^{(+)}_{++}(0;\tau)\right|^2 + \left|B^{(-)}_{++}(0;\tau)\right|^2 \\ & + \left|B^{(+)}_{+0}(1;\tau)\right|^2 + \left|B^{(-)}_{+0}(1;\tau)\right|^2 \\ & + \left|B^{(+)}_{+-}(2;\tau)\right|^2 + \left|B^{(-)}_{+-}(2;\tau)\right|^2 \end{aligned} \tag{22}$$

Consider now an approximation in which *both* photons are on-shell, even though one of them has been 'tagged.' Then the distribution function consists of four terms only; the first two plus the last two terms in (22). Suppose, in addition, that spins greater than 2 can be neglected near threshold, e.g. $\sqrt{s} < 3.0$ GeV. The distribution function may be written

$$\begin{aligned} I(\tau) = {} & \left|A_{++}(0^{++}\,0;\tau) + A_{++}(1^{-+}\,0;\tau) + A_{++}(2^{++}\,0;\tau)\right|^2 \\ & + \left|A_{++}(0^{-+}\,0;\tau) + A_{++}(1^{++}\,0;\tau) + A_{++}(2^{-+}\,0;\tau)\right|^2 \\ & + \left|A_{+-}(2^{++}\,2;\tau)\right|^2 + \left|A_{+-}(2^{-+}\,2;\tau)\right|^2 \end{aligned} \tag{23}$$

Two spin-1 states, $|1^{++}\,0\rangle$ and $|1^{-+}\,0\rangle$, appear in the formula above; the latter is an exotic state. Note, in particular, that the two spin-1 states are produced coherently with the spin-0 and spin-2 states of the same naturality.

3 Kinematics and Phase Space

In this section, we give a general overview of the phenomenological formalism[4] necessary for the treatment of the processes

$$\begin{aligned} &e^+(p_a) + e^-(p_b) \to e^+(p'_a) + e^-(p'_b) + \gamma_1(p_1) + \gamma_2(p_2) \\ &\gamma_1(p_1) + \gamma_2(p_2) \to X(p) \end{aligned} \tag{24}$$

where the appropriate 4-momenta are given in parentheses. We shall use a convention in which *both* the 4-momenta *and* the magnitude of the 3-momenta are given the same notation. Let E_a and E_b the beam energies of e^+ and e^-in the collider frame; then one has $E_a = E_b$ and $p_a = p_b$, and one finds $E_a = p_a$ and $E_b = p_b$ in the approximation $m_e \approx 0$. Emitted photon energies are denoted ω_1 and ω_2. The relevant quantities of interest are

$$\begin{aligned} &p_a = p'_a + p_1, \quad \omega_1 = E_a - E'_a, \quad t_1 = p_1^2 = \omega_1^2 - p_{1L}^2 - p_{1T}^2 \\ &p_b = p'_b + p_2, \quad \omega_2 = E_b - E'_b, \quad t_2 = p_2^2 = \omega_2^2 - p_{2L}^2 - p_{2T}^2 \\ &S = (p_a + p_b)^2 = 4E_a\, E_b = 4E_a^2 = 4p_a^2 \\ &s = W^2 = p^2 = E^2 - p_L^2 - p_T^2, \quad \mathbf{p}_T = \mathbf{p}_{1T} + \mathbf{p}_{2T} \\ &\quad = (p_1 + p_2)^2 \\ &\quad = t_1 + t_2 + 2(\omega_1\, \omega_2 + p_{1L} p_{2L} - \mathbf{p}_{1T} \cdot \mathbf{p}_{2T}) \end{aligned} \tag{25}$$

where the subscripts L indicate the longitudinal components along the positive or the negative z-axis (defined to be along the e^+ beam direction), and the subscripts T stand for the 2-dimensional vectors transverse to the beam. Note that

$$\mathbf{p}'_{aT} + \mathbf{p}_{1T} = 0, \qquad \mathbf{p}'_{bT} + \mathbf{p}_{2T} = 0, \qquad \mathbf{p}'_{aT} + \mathbf{p}'_{bT} + \mathbf{p}_T = 0$$

See Fig. 1 for an illustration of the coordinate system used for the reaction.

Several useful relationships[4] are derived here. Define ϵ's through the magnitude of the 3-momenta

$$\begin{aligned} p'_a &= (1 - \epsilon_1) p_a \\ p'_b &= (1 - \epsilon_2) p_b \end{aligned} \tag{26}$$

Evidently, one must have $0 < \epsilon_1 < 1$ and $0 < \epsilon_2 < 1$. In the approximation $m_e \approx 0$, one has

$$\begin{aligned} E'_a &= (1 - \epsilon_1) E_a \\ E'_b &= (1 - \epsilon_2) E_b \end{aligned} \tag{27}$$

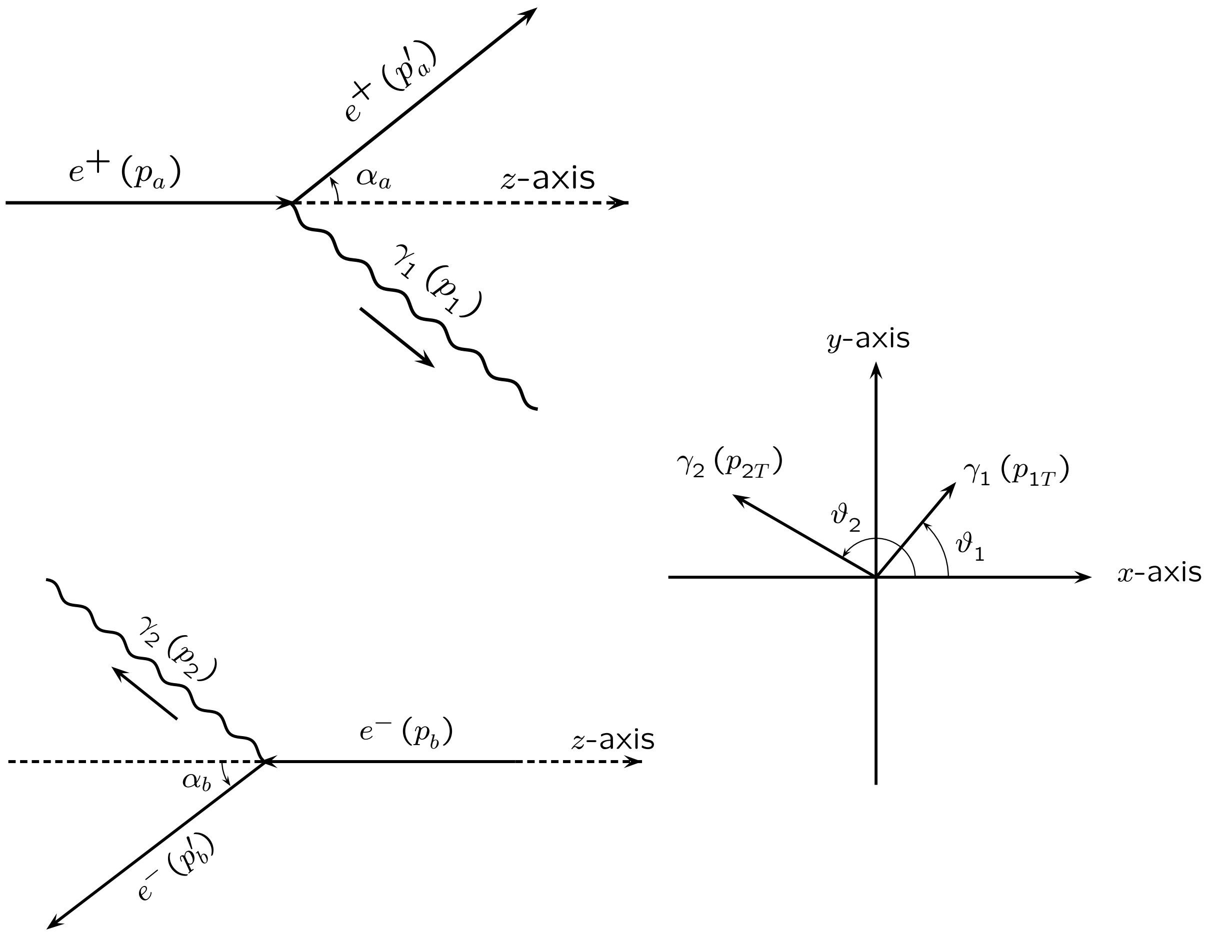

Figure 1: The process for $e^+ + e^- \to e^+ + e^- + \gamma_1 + \gamma_2$. The subprocesses $e^+ \to e^+ + \gamma_1$ and $e^- \to e^- + \gamma_2$ are each coplanar. The angle between the planes is given by $\vartheta_2 - \vartheta_1$.

so that $\epsilon_1 = \omega_1/E_a$ and $\epsilon_2 = \omega_2/E_b$. Note that ϵ's thus reflect the fractional energy loss due to photon emission. There is one instance in which the approximation $m_e \approx 0$ cannot be used; it occurs in the expressions for the momentum transfers. Since one has

$$E_a \simeq p_a + \frac{m_e^2}{2p_a}$$
$$E'_a \simeq p'_a + \frac{m_e^2}{2p'_a} = p_a\,(1-\epsilon_1) + \frac{m_e^2}{2p_a\,(1-\epsilon_1)}$$

and similarly for E_b and E_b', the momentum transfers are given by [a]

$$
\begin{aligned}
-t_1 &\simeq (-t_1)_{\min} + 4p_a\, p_a' \,\sin^2\left(\frac{\alpha_a}{2}\right) \\
&\simeq -2m_e^2 + 2(E_a\, E_a' - p_a\, p_a') + p_a\, p_a'\, \alpha_a^2 \simeq \frac{(m_e\, \epsilon_1)^2}{1-\epsilon_1} + \frac{p_{1T}^2}{1-\epsilon_1} \\
-t_2 &\simeq (-t_2)_{\min} + 4p_b\, p_b' \,\sin^2\left(\frac{\alpha_b}{2}\right) \\
&\simeq -2m_e^2 + 2(E_b\, E_b' - p_b\, p_b') + p_b\, p_b'\, \alpha_b^2 \simeq \frac{(m_e\, \epsilon_2)^2}{1-\epsilon_2} + \frac{p_{2T}^2}{1-\epsilon_2}
\end{aligned}
\tag{28}
$$

The angles α_a and α_b are the scattering angles (small) in the collider frame. We have put [b] $p_{1T} \simeq p_a'\, \alpha_a$ and $p_{2T} \simeq p_b'\, \alpha_b$.

The effective mass W can be expressed as, in the approximation $m_e \approx 0$,

$$
\begin{aligned}
W^2 &= (p_1 + p_2)^2 = (\omega_1 + \omega_2)^2 - (\mathbf{p}_a - \mathbf{p}_a' + \mathbf{p}_b - \mathbf{p}_b')^2 \\
&= 4\omega_1\omega_2 + (\omega_1 - \omega_2)^2 - (p_{aL}' - p_{bL}')^2 - (\mathbf{p}_{aT}' + \mathbf{p}_{bT}')^2 \\
&= 4\omega_1\omega_2 + (E_a' - E_b')^2 - (p_a')^2 - (p_b')^2 \\
&\qquad + 2p_a'p_b' \,[\cos\alpha_a\, \cos\alpha_b - \sin\alpha_a\, \sin\alpha_b\, \cos(\vartheta_1 - \vartheta_2)] \\
&= 4\omega_1\omega_2 - 2p_a'p_b' \,[1 - \cos\alpha_a\, \cos\alpha_b + \sin\alpha_a\, \sin\alpha_b\, \cos(\vartheta_1 - \vartheta_2)] \\
&= \omega_1\omega_2 - 2\, E_a^2\, (1-\epsilon_1)\, (1-\epsilon_2)\, (1 - \cos\alpha_a\, \cos\alpha_b) - 2\, p_{1T}\, p_{2T}\, \cos(\vartheta_1 - \vartheta_2)
\end{aligned}
\tag{29}
$$

where the angles ϑ_1 and ϑ_2 specify the direction of the vectors $\mathbf{p}_{1T}$ and $\mathbf{p}_{2T}$ in the transverse plane. The formula above shows that $W^2 \simeq 4\omega_1\omega_2$, if the angles α_a and α_b are sufficiently small. We now wish to find a alternative relationship between W and the ϵ's defined above. Noting that [see (24)]

$$
\begin{aligned}
E &= E_a + E_b - E_a' - E_b' \\
p_L &= +p_{aL}' - p_{bL}'
\end{aligned}
$$

and

$$
\begin{aligned}
p_{aL}' &= p_a' \sqrt{1 - \alpha_a^2} \simeq (1-\epsilon_1)\, p_a - \frac{p_{1T}^2}{2(1-\epsilon_1)\, p_a} \\
p_{bL}' &= p_b' \sqrt{1 - \alpha_b^2} \simeq (1-\epsilon_2)\, p_a - \frac{p_{2T}^2}{2(1-\epsilon_2)\, p_a}
\end{aligned}
$$

---

[a] The reader will note that the factor $(1-\epsilon_i)^{-1}$ is missing in Ref.[4], but this is nearly equal to 1 in the approximation considered there. We retain this factor for the first term to conform to the formula given in Ref.[6].

[b] The factor $(1-\epsilon_i)^{-1}$ for the second term in (28) comes naturally from this definition.

one finds

$$\begin{aligned} E &\simeq (\epsilon_1+\epsilon_2)p_a \\ p_{\scriptscriptstyle L} &\simeq (-\epsilon_1+\epsilon_2)p_a - \frac{p_{1T}^2}{2(1-\epsilon_1)\,p_a} + \frac{p_{2T}^2}{2(1-\epsilon_2)\,p_a} \end{aligned} \tag{30}$$

These relationships enable one to express the transverse mass of X as

$$\begin{aligned} W_T^2 = W^2 + p_T^2 &= (E+p_{\scriptscriptstyle L})(E-p_{\scriptscriptstyle L}) \\ &\simeq S\,\epsilon_1\,\epsilon_2 - \left(\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}\right) p_{1T}^2 - \left(\frac{\epsilon_2-\epsilon_1}{1-\epsilon_2}\right) p_{2T}^2 \end{aligned} \tag{31}$$

neglecting higher-order terms in p_{1T}^2 and p_{2T}^2. Since one has

$$p_T^2 = p_{1T}^2 + p_{2T}^2 + 2\,p_{1T}\,p_{2T}\,\cos(\vartheta_1-\vartheta_2) \tag{32}$$

or

$$\xi = \kappa + v_1^2 + v_2^2 + 2\,v_1\,v_2\,\cos(\vartheta_1-\vartheta_2) \tag{33}$$

one sees that

$$\xi \simeq \epsilon_1\,\epsilon_2 - (\epsilon_1-\epsilon_2)\left(\frac{v_1^2}{1-\epsilon_1} - \frac{v_2^2}{1-\epsilon_2}\right) \tag{34}$$

where $\xi = W_T^2/S$, $\kappa = W^2/S$, $v_1^2 = p_{1T}^2/S$ and $v_2^2 = p_{2T}^2/S$. Eliminating ξ from (33) and (34), one sees that

$$\kappa \simeq \epsilon_1\epsilon_2 - \left(\frac{1-\epsilon_2}{1-\epsilon_1}\right) v_1^2 - \left(\frac{1-\epsilon_1}{1-\epsilon_2}\right) v_2^2 - 2\,v_1\,v_2\,\cos(\vartheta_1-\vartheta_2) \tag{35}$$

It can be shown that the formula above can be obtained directly from (29) by retaining up to second-order terms in α_a and α_b.

The rapidity is given in the usual way

$$\begin{aligned} y &= \tanh^{-1}\left(\frac{p_{\scriptscriptstyle L}}{E}\right) = \frac{1}{2}\ln\left(\frac{E+p_{\scriptscriptstyle L}}{E-p_{\scriptscriptstyle L}}\right) \\ &\simeq \frac{1}{2}\ln\left(\epsilon_2 - \frac{v_1^2}{1-\epsilon_1} + \frac{v_2^2}{1-\epsilon_2}\right) - \frac{1}{2}\ln\left(\epsilon_1 + \frac{v_1^2}{1-\epsilon_1} - \frac{v_2^2}{1-\epsilon_2}\right) \end{aligned} \tag{36}$$

It can be shown that

$$\begin{aligned} \epsilon_1 &\simeq \sqrt{\xi}\,\mathrm{e}^{-y} - \frac{v_1^2}{1-\epsilon_1} + \frac{v_2^2}{1-\epsilon_2} \\ \epsilon_2 &\simeq \sqrt{\xi}\,\mathrm{e}^{\,y} + \frac{v_1^2}{1-\epsilon_1} - \frac{v_2^2}{1-\epsilon_2} \end{aligned} \tag{37}$$

neglecting higher-order terms in v_1^2 and v_2^2. In order to proceed further, one needs to make an *additional* assumption that ϵ_1 and ϵ_2 are small compared to 1. Since v_1^2 and v_2^2 are already small, this restriction on ϵ_1 and ϵ_2 is not very strigent. We merely require that they be far from 1. Note that this is equivalent to a restriction on the size of the rapidity y; it implies that y cannot be arbitrarily large. Replacing $(1-\epsilon_i)^{-1}$ by $(1+\epsilon_i)$ and solving for ϵ_1 and ϵ_2, one finds the simple result

$$\begin{aligned}
\epsilon_1 &\simeq \sqrt{\xi}\, \mathrm{e}^{-y}\,(1-v_1^2) + \sqrt{\xi}\, \mathrm{e}^{\,y}\, v_2^2 - v_1^2 + v_2^2 \\
\epsilon_2 &\simeq \sqrt{\xi}\, \mathrm{e}^{\,y}\,(1-v_2^2) + \sqrt{\xi}\, \mathrm{e}^{-y}\, v_1^2 + v_1^2 - v_2^2
\end{aligned} \tag{38}$$

It should be noted that these formulas give $\epsilon_1 + \epsilon_2 = \sqrt{\xi}\,(\mathrm{e}^{\,y} + \mathrm{e}^{-y})$, as do those of (37).

The evaluation of the invariant phase-space element for the process $e^+e^- \to e^+e^- X$ requires a factor

$$\mathrm{d}\Phi = \frac{\mathrm{d}^3\mathbf{p}'_a}{2E'_a}\,\frac{\mathrm{d}^3\mathbf{p}'_b}{2E'_b} = \frac{1}{4}\,\frac{\mathrm{d}p'_{aL}}{E'_a}\,\frac{\mathrm{d}p'_{bL}}{E'_b}\,\mathrm{d}^2\mathbf{p}_{1T}\,\mathrm{d}^2\mathbf{p}_{2T} \tag{39}$$

where all other revelant factors, including the δ-function expressing the four-momentum conservation, have been absorbed into the formula for the two-photon cross section. From $\omega_1 = E_a - E'_a$ and $\omega_2 = E_a - E'_b$, one finds that

$$\begin{aligned}
\mathrm{d}\Phi &= \frac{1}{4}\,\frac{\mathrm{d}\omega_1}{p'_{aL}}\,\frac{\mathrm{d}\omega_2}{p'_{bL}}\,\mathrm{d}^2\mathbf{p}_{1T}\,\mathrm{d}^2\mathbf{p}_{2T} \\
&= \frac{1}{4}\left(\frac{\mathrm{d}\epsilon_1}{1-\epsilon_1}\right)\left(\frac{\mathrm{d}\epsilon_2}{1-\epsilon_2}\right)\mathrm{d}^2\mathbf{p}_{1T}\,\mathrm{d}^2\mathbf{p}_{2T} \\
&= \frac{1}{16}\left(\frac{\mathrm{d}\epsilon_1}{1-\epsilon_1}\right)\left(\frac{\mathrm{d}\epsilon_2}{1-\epsilon_2}\right)\mathrm{d}p_{1T}^2\,\mathrm{d}p_{2T}^2\,\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2 \\
&= \frac{1}{16}\,\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2\,\mathrm{d}t_1\,\mathrm{d}t_2\,\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2
\end{aligned} \tag{40}$$

For the application considered here, one needs to transform the phase-space variables into those involving the effective mass W for X. Observe that

$$\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2 \simeq (1 - v_1^2 - v_2^2)\,\mathrm{d}y\,\mathrm{d}\xi \tag{41}$$

evaluating, from (38), the appropriate Jacobian. One obtains[c]

$$\begin{aligned}
\mathrm{d}\Phi &= \left(\frac{t_1\,t_2}{16}\right)\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2\left(\frac{\mathrm{d}t_1}{t_1}\right)\left(\frac{\mathrm{d}t_2}{t_2}\right)\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2 \\
&\simeq \left(\frac{t_1\,t_2}{16}\right)(1 - v_1^2 - v_2^2)\,\mathrm{d}y\,\mathrm{d}\kappa\left[\frac{\mathrm{d}p_{1T}^2}{(m_e\,\epsilon_1)^2 + p_{1T}^2}\right]\left[\frac{\mathrm{d}p_{2T}^2}{(m_e\,\epsilon_2)^2 + p_{2T}^2}\right]\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2
\end{aligned} \tag{42}$$

[c] The reader will note that the phase-space formula given here can be obtained from a very different approach, as in Ref.[4]. It is seen that the factors $(1-\epsilon_1)^{-1}$ and $(1-\epsilon_2)^{-1}$ are missing in the formula in the cited reference. However, the difference is minor, since ϵ's are rather small in most applications.

We do not integrate over these angles ϑ_1 and ϑ_2, because p_T^2 given by (32) enters into the definition of ξ and hence it must be held fixed [see (37)]. Note that the transition from $\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2 \to \mathrm{d}y\,\mathrm{d}\xi \to \mathrm{d}y\,\mathrm{d}\kappa$ in (42) can occur only when $\mathbf{p}_{1T}$ and $\mathbf{p}_{2T}$, and hence p_T^2, remain specified.

4 Differential Cross Sections

According to V. M. Budnev *et al.*[7], the cross section for two-photon processes at LEP, $e^+e^- \to e^+e^-X$ with $\gamma\gamma \to X$, can be written

$$\mathrm{d}\sigma(e^+e^- \to e^+e^-X; S) = \mathrm{d}N_1\,\mathrm{d}N_2\,\mathrm{d}\sigma(\gamma\gamma \to X; s) \tag{43}$$

where N_1 and N_2 are the numbers of photons with energies ω_1 and ω_2 and momentum-transfers t_1 and t_2, respectively. We have adopted an approximation in which the polarization of the virtual photons is ignored. The photon distribution function F is given via

$$\mathrm{d}N_i = F(t_i, \epsilon_i)\left(\frac{\mathrm{d}\vartheta_i}{2\pi}\right)\left(\frac{\mathrm{d}\epsilon_i}{\epsilon_i}\right)\left(\frac{\mathrm{d}t_i}{t_i}\right) \tag{44}$$

for $i = 1$ or 2. The functions $F(t_i, \epsilon_i)$ are

$$F(t_i, \epsilon_i) = \left(\frac{\alpha}{\pi}\right)\left(\frac{\rho_i^{++}}{2}\right)\epsilon_i^2 \tag{45}$$

where

$$\begin{aligned} 2\,\rho_1^{++} &= \frac{(2p_a \cdot p_2 - p_1 \cdot p_2)^2}{X} + \frac{4m_e^2}{t_1} + 1 \\ 2\,\rho_2^{++} &= \frac{(2p_b \cdot p_1 - p_1 \cdot p_2)^2}{X} + \frac{4m_e^2}{t_2} + 1 \end{aligned} \tag{46}$$

Noting that, in the approximation $m_e \approx 0$,

$$\begin{aligned} 2p_a \cdot p_2 &= (p_a + p_2)^2 - p_a^2 - p_2^2 = (p_a + p_b - p_b')^2 - t_2 \\ 2p_b \cdot p_1 &= (p_b + p_1)^2 - p_b^2 - p_1^2 = (p_b + p_a - p_a')^2 - t_1 \end{aligned}$$

one finds

$$\begin{aligned} 2p_a \cdot p_2 &= 4E_a\omega_2 - t_2 \\ 2p_b \cdot p_1 &= 4E_a\omega_1 - t_1 \\ 2p_1 \cdot p_2 &= W^2 - t_1 - t_2 \end{aligned}$$

so that

$$
\begin{aligned}
F(t_1,\epsilon_1) &= \left(\frac{\alpha}{\pi}\right)\left[\frac{(2\,S\,\epsilon_2 - W^2 + t_1 - t_2)^2}{16\,X} + \frac{m_e^2}{t_1} + \frac{1}{4}\right]\,\epsilon_1^2 \\
F(t_2,\epsilon_2) &= \left(\frac{\alpha}{\pi}\right)\left[\frac{(2\,S\,\epsilon_1 - W^2 + t_2 - t_1)^2}{16\,X} + \frac{m_e^2}{t_2} + \frac{1}{4}\right]\,\epsilon_2^2
\end{aligned}
\tag{47}
$$

where

$$
4\,X = W^4 - 2\,W^2\,(t_1 + t_2) + (t_1 - t_2)^2 \tag{48}
$$

and W^2 is given either by (29) for arbitrary scattering angles and by (35) for small scattering angles.

If $-t_i \ll W^2$ (or small scattering angles), then the photon distribution functions can be simplified greatly. Dropping the t_i's in the numerators of (47) and substituting $W^2 \to S\,\epsilon_1\epsilon_2$ and $4\,X \to (S\,\epsilon_1\epsilon_2)^2$, one can obtain the familiar result

$$
F(t_i,\epsilon_i) = \left(\frac{\alpha}{\pi}\right)\,\left[1 - \epsilon_i + \left(\frac{1}{2} + \frac{m_e^2}{t_i}\right)\epsilon_i^2\right] \tag{49}
$$

It is convenient to recast this formula

$$
\frac{\mathrm{d}N_i}{\mathrm{d}\epsilon_i\,\mathrm{d}u_i^2} = G(u_i,\epsilon_i)\,\left(\frac{1}{\epsilon_i^2 + u_i^2}\right)\,\left(\frac{\mathrm{d}\vartheta_i}{2\pi}\right) \tag{50}
$$

where $F = \epsilon_i\,G$, so that

$$
G(u_i,\epsilon_i) = \left(\frac{\alpha}{\pi}\right)\,\left[\frac{1-\epsilon_i}{\epsilon_i} + \frac{\epsilon_i}{2} - \frac{(1-\epsilon_i)\,\epsilon_i}{\epsilon_i^2 + u_i^2}\right] \tag{51}
$$

and $u_i = p_{iT}/m_e$. Now the formula has been given entirely in unitless variables. This formula takes on a particularly simple form at $u_i = 0$. Noting that the dependence on ϑ_i disappears in this limit, one finds

$$
\left[\frac{\mathrm{d}N_i}{\mathrm{d}\epsilon_i\,\mathrm{d}u_i^2}\right]_{u_i=0} = \left(\frac{\alpha}{2\pi}\right)\,\frac{1}{\epsilon_i} \tag{52}
$$

It should be emphasized that the formula above can be derived directly from the general one given in (47). As long as ϵ_i remains finite, the photon flux is nonzero in the limit $u_i = 0$.

The differential cross section can be cast into

$$
\begin{aligned}
\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^- X; S)}{\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2\,\mathrm{d}\tau} \simeq \int F(t_1,\epsilon_1)\,F(t_2,\epsilon_2)\,\left(\frac{\mathrm{d}\vartheta_1}{2\pi}\right)\,\left(\frac{\mathrm{d}\vartheta_2}{2\pi}\right)\,\left(\frac{\mathrm{d}t_1}{\epsilon_1\,t_1}\right)\,\left(\frac{\mathrm{d}t_2}{\epsilon_2\,t_2}\right) \\
\times\,\frac{\mathrm{d}\sigma(\gamma\gamma \to X; s)}{\mathrm{d}\tau}
\end{aligned}
\tag{53}
$$

where $g(\tau)\,\mathrm{d}\tau$ is the invariant phase-space element given by the final state which the X decays into. Using (42), the formula can be rewritten[d]

$$\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^- X; S)}{\mathrm{d}y\,\mathrm{d}\kappa\,\mathrm{d}\tau} \simeq H_0(z_1, z_2, y, \kappa)\, \frac{\mathrm{d}\sigma(\gamma\gamma \to X; s)}{\mathrm{d}\tau} \tag{54}$$

where

$$H_0(z_1, z_2, y, \kappa) = \int (1 - v_1^2 - v_2^2)\, G(u_1, \epsilon_1)\, G(u_2, \epsilon_2) \left(\frac{\mathrm{d}u_1^2}{\epsilon_1^2 + u_1^2}\right) \left(\frac{\mathrm{d}u_2^2}{\epsilon_2^2 + u_2^2}\right) \left(\frac{\mathrm{d}\vartheta}{2\pi}\right) \tag{55}$$

and $z_i = u_{i\,\max} = p_{iT\,\max}/m_e$. The range of integration is $(0 \to 2\pi)$ for ϑ and $(0 \to z_i^2)$ for u_i^2, where $p_{1T\,\max} \simeq p'_a\,\alpha_{\max}$ and $p_{2T\,\max} \simeq p'_b\,\alpha_{\max}$. ($\alpha_{\max}$ is the maximum for α_a, fixed by the collider and detector geometry.) It is clear that an accurate evaluation of H_0 calls for substituting G in (55) by $G = F/\epsilon_i$ with the F given in (47) together with (48) and (29).

Most of the literature on two-photon processes deal with the case in which $W \gg p_T$, leading to a simple closed form

$$\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^- X; S)}{\mathrm{d}y\,\mathrm{d}\kappa\,\mathrm{d}\tau} \simeq H(z_1, \epsilon_1)\, H(z_2, \epsilon_2)\, \frac{\mathrm{d}\sigma(\gamma\gamma \to X; s)}{\mathrm{d}\tau} \tag{56}$$

where

$$H(z_i, \epsilon_i) = \left(\frac{\alpha}{\pi}\right) \left[\left(\frac{1-\epsilon_i}{\epsilon_i} + \frac{\epsilon_i}{2}\right) \ln\left(1 + z_i^2/\epsilon_i^2\right) - \frac{1-\epsilon_i}{\epsilon_i}\left(1 - \frac{1}{1 + z_i^2/\epsilon_i^2}\right)\right] \tag{57}$$

However, in an application in which a study is made of the mesons in the mass range comparable to p_T, one has to resort to the formula (54). This is illustrated in Figs.2 and 3. The two-photon flux is nearly flat as a function of rapidity, as shown in 4.

[d] We can set $\vartheta_2 = 0$ and then integrate over ϑ_2 in (32), so that p_T^2 is a function of $\vartheta \equiv \vartheta_1$ only. Note that the remaining integration is over three variables: ϑ, $u_1^2 = (p_{1T}/m_e)^2$ and $u_2^2 = (p_{2T}/m_e)^2$.

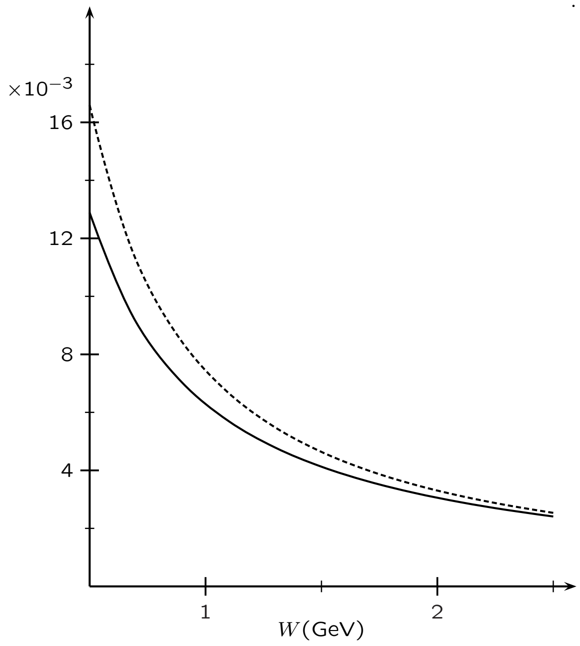

Figure 2: Two-photon flux for $y = 0$ with the mass W in the range $0.5 < W < 2.5$ GeV. The solid curve corresponds to $H_0(z_1, z_2, y, \kappa)$ [see (55)] and the dashed curve to $H(z_1, \epsilon_1)\, H(z_2, \epsilon_2)$ [see (57)]. We have used $\sqrt{S} = 175$ GeV and $\alpha_{\rm max} = 30$ mrad.

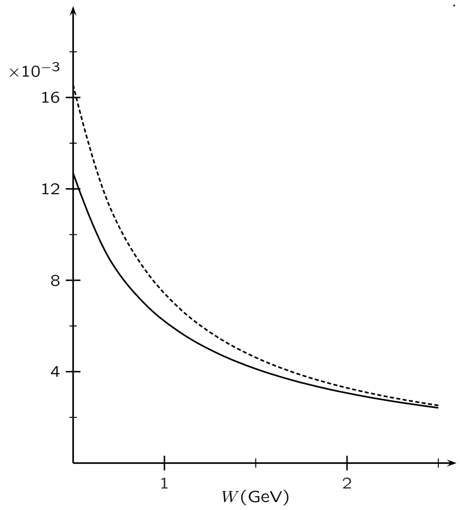

Figure 3: Same as Fig.2, except that the rapidity is $y = \pm 1$.

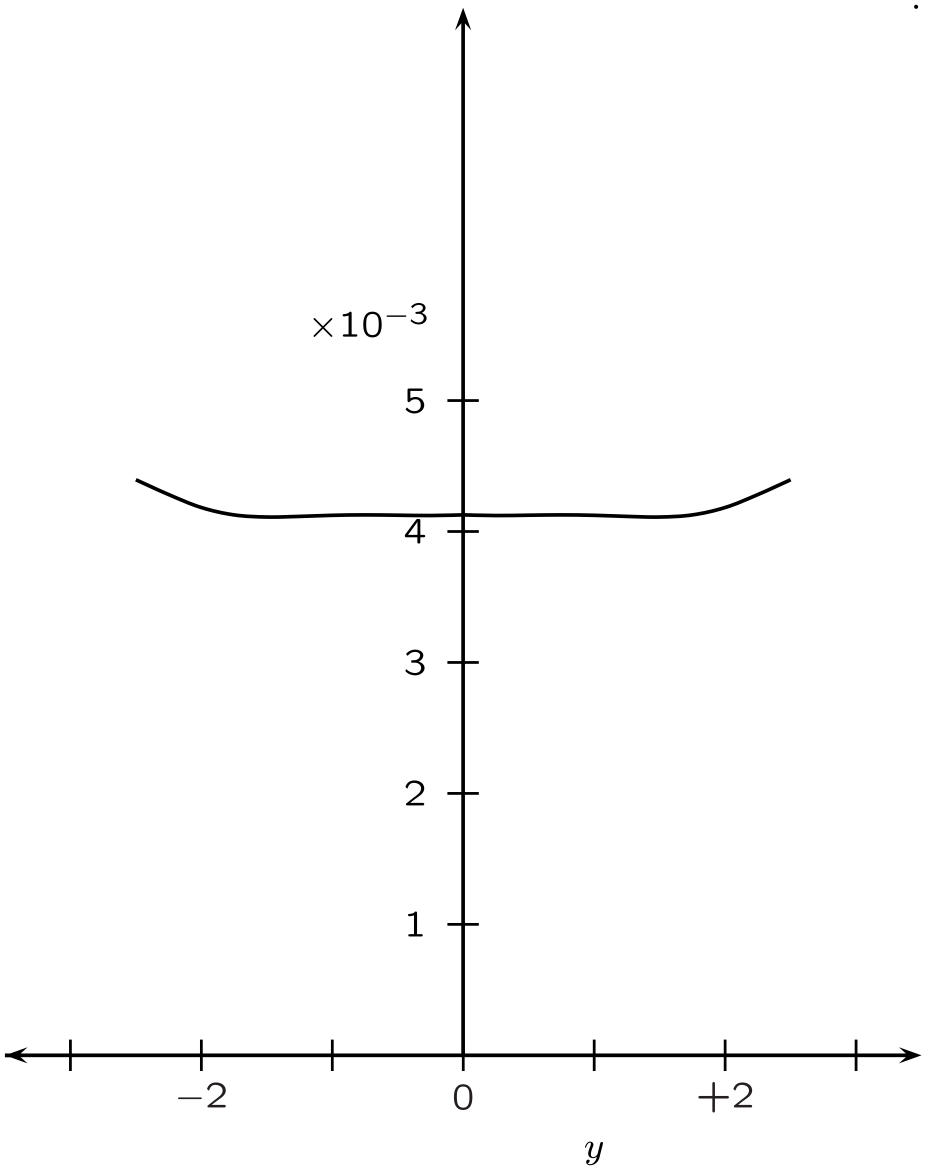

Figure 4: Two-photon flux as a function of rapidity at $W = 1.50$ GeV.

5 Two-Photon Decay Amplitudes

We consider here production of a given J^{PC} state in the s-channel in the reaction

$$\gamma_1\gamma_2 \to \rho_1^0\rho_2^0 \to \pi_1^+\pi_2^-\pi_3^+\pi_4^- \tag{58}$$

where $\rho_1^0 \to \pi_1^+\pi_2^-$ and $\rho_2^0 \to \pi_3^+\pi_4^-$. The reaction is illustrated in Fig. 5. Note that $C = +1$

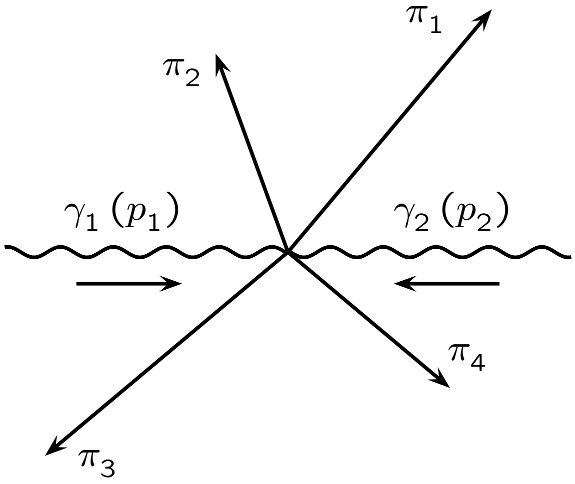

Figure 5: The process $\gamma_1\gamma_2 \to \pi_1^+\pi_2^-\pi_3^+\pi_4^-$.

for the 4π system since it results from the two-photon initial system. Since the 4π system must have $G = +1$, it should have $I = 0$ or $I = 2$. So, most likely, the 4π system comes the quantum numbers $I^G\,(J^{PC}) = 0^+\,(J^{P\,+})$, including the J^{PC} exotic mesons which could be produced in the s-channel in (58).

The relativistic phase-space element $g(\tau)\,\mathrm{d}\tau$, appropriate for reaction (58),

$$\begin{aligned} g(\tau) &= \frac{p_\rho}{W}\,k_1\,k_2 \\ \mathrm{d}\tau &= \mathrm{d}w_1\,\mathrm{d}w_2\,\mathrm{d}\cos\theta\,\mathrm{d}\Omega_1\,\mathrm{d}\Omega_2 \end{aligned} \tag{59}$$

where $W = \sqrt{s}$ is the effective mass of the $\rho^0\rho^0$ system, equal to the center-of-mass (CM) for the reaction (58); p_ρ is the CM breakup momentum of the $\rho^0\rho^0$ system; $w_1(w_2)$ is the effective mass of the $\pi_1^+\pi_2^-(\pi_3^+\pi_4^-)$; $k_1(k_2)$ is the breakup momentum of the $\pi_1^+\pi_2^-(\pi_3^+\pi_4^-)$ system in its rest frame; θ is the polar angle of the $\rho_1^0 = \pi_1^+ + \pi_2^-$ with respect to the $\gamma\gamma$ axis (the corresponding azimuthal angle can be set to zero, without loss of generality); $\Omega_1 = (\theta_1, \phi_1)$

describes the direction of π_1^+ in the helicity coordinate system[2] defined in the rest frame of the ρ_1^0; and, likewise, $\Omega_2 = (\theta_2, \phi_2)$ describes the direction of π_3^+ in the rest frame of the ρ_2^0. The differential cross section for the process (58) can be written

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \propto \frac{1}{W^2}\, I(\tau)\, g(\tau) \tag{60}$$

where τ represents a set of 7 variables as specified in (59). We have fixed the flux factor by $\mathcal{F} = p_\gamma W = W^2/2$, assuming that the cross section formula above refers to the case in which the photons are massless, i.e. $p_\gamma = W/2$.

The amplitudes discussed in Section 3 can now be given

$$\begin{aligned} A(J^{PC}\Lambda;\tau) = &\left(\frac{2J+1}{4\pi}\right)^{1/2}\left(\frac{2s_1+1}{4\pi}\right)^{1/2}\left(\frac{2s_2+1}{4\pi}\right)^{1/2} \\ &\times \sum_{\mu_1\mu_2} G^a_{\mu_1\mu_2}\, d^J_{\Lambda,\mu_1-\mu_2}(\theta)\, D^{s_1\,*}_{\mu_1,0}(\phi_1,\theta_1,0)\, D^{s_2\,*}_{\mu_2,0}(\phi_2,\theta_2,0) \end{aligned} \tag{61}$$

where $s_1 = s_2 = 1$ stands for the ρ spin, and μ_1 and μ_2 are the helicities of the ρ_1^0 and ρ_2^0, respectively. $G^a_{\mu_1\mu_2}$ is the helicity-coupling amplitude for the $\rho^0\rho^0$ system, the equivalent of the F^a's for $\gamma\gamma$ states considered in Section 3.

The helicity-coupling amplitudes G^a's can be expressed

$$\begin{aligned} G^a_{\mu_1\mu_2} = &(k_1)^{s_1}\,\Delta(w_1)\,(k_2)^{s_2}\,\Delta(w_2) \\ &\times \sum_{LS}\left(\frac{2L+1}{2J+1}\right)^{1/2} p_\rho^L\, R^a_{LS}\,(L0\,S\delta|J\delta)\,(s_1\mu_1\,s_2\,{-\mu_2}|S\delta) \end{aligned} \tag{62}$$

where $\delta = \mu_1 - \mu_2$. $\Delta(w)$ is a relativistic Breit-Wigner form for the ρ; p_ρ^L, $(k_1)^{s_1}$ and $(k_2)^{s_2}$ are the appropriate angular-momentum barrier factors, which are replaced in practice by the Blatt-Weisskopf form factors[8]; the usual Clebsch-Gordan coefficients are denoted $(j_1m_1j_2m_2|jm)$; and R^a_{LS} is the parameter to be determined from data. For each given $a = J^{PC}$, there exists a set of allowed $\{L,S\}$, i.e. $\eta_a = (-)^L$, $|L-S| \le J \le L+S$ and $\{S = 0,1,2\}$, with a contraint that $L+S =$ even, which comes from the presence of identical ρ^0's in the intermediate state. The photon helicities $\{\lambda_1\,\lambda_2\}$ have been suppressed from the amplitude A, $G^a_{\mu_1\mu_2}$ and R^a_{LS}, respectively, for ease of notation.

We note that the formula (62) is non-relativistic. It lacks the functional dependence on the Lorentz factors, $\gamma_1 = E_1/w_1$ and $\gamma_2 = E_2/w_2$, which must be present, since the intermediate states ρ_1 and ρ_2 have finite momenta in the two-photon center-of-mass system. Essentially, the decay amplitudes in general must depend not only on the momenta p_ρ, k_1 and k_2 but also on the energies E_1 and E_2 defined in the overall rest frame. The reader may consult Ref.[9] for the general prescription of such a relativistic formalism.

Finally, a comment is in order on the shape of the W spectrum. For the purpose, consider the integrated cross section

$$\sigma(W) \propto \frac{1}{W^2} \int I(\tau)\, g(\tau)\, \mathrm{d}\tau \tag{63}$$

It is clear that $\sigma(W)$ as a function of W has no physical meaning, i.e. the formalism given here cannot say anything about the shape of the W spectrum. It is given instead by the convolution of the two-photon physical process, which we wish to explore, and the experimental acceptance.

Acknowledgement

I am indebted to D. Ryabchikov/IHEP(Protvino) for several helpful discussions.

Appendix A

Consider a process

$$\gamma + \gamma \to R(\,^{2S+1}L_J) \to e^+ + e^- \tag{64}$$

where R is a quarkonium state, i.e. a bound state of $q + \bar{q}$, communicating with $\gamma + \gamma$ and $e^+ + e^-$. A concise summary of the two-photon helicity selection rules[10] is given here.

Because of the $C = +1$ for an R coupling to $\gamma + \gamma$, one must have either $S = 0$ and $L =$ even or $S = 1$ and $L =$ odd. Let the reduced partial width be denoted by $\hat{\Gamma}_\Lambda$ where Λ is the net helicity of the two on-shell photons, so that $\Lambda = 0$ or 2. The results are shown in Table II.

Table II: Reduced Partial Widths

J^{PC}	S	L	J	Λ	$\hat{\Gamma}_\Lambda$
even^{-+}	0	$L=$ even	$J=L$	0	1
even^{-+}	0	$L=$ even	$J=L$	2	0
even^{++}	1	$L=$ odd	$J=L+1$	0	0
even^{++}	1	$L=$ odd	$J=L+1$	2	$(L+2)(L+3)\Big/L(2L+3)$
odd^{++}	1	$L=$ odd	$J=L$	0	0
odd^{++}	1	$L=$ odd	$J=L$	2	$(L-1)(L+2)\Big/L^2$
even^{++}	1	$L=$ odd	$J=L-1$	0	$(2L+1)^2\Big/L(2L-1)$
even^{++}	1	$L=$ odd	$J=L-1$	2	$f^2\,(L-2)(L-1)(L+1)\Big/L^2(2L-1)$

where

$$f = \begin{cases} 1, & L=1; \\ 1+\Big\{L(2L+1)\Big/2(L-2)(L-1)\Big\}, & L\geq 3 \end{cases} \tag{65}$$

It is probably more useful to write down the results explicitly for $L = 0$, 1, 2 and 3. The results are summarized in Table III.

Table III: Reduced Partial Widths for $L \leq 3$

J^{PC}	${}^{2S+1}L_J$	Λ	$\hat{\Gamma}_\Lambda$
0^{-+}	${}^1S_0$	0	1
0^{-+}	${}^1S_0$	2	0
0^{++}	${}^3P_0$	0	9
0^{++}	${}^3P_0$	2	0
2^{++}	${}^3P_2$	0	0
2^{++}	${}^3P_2$	2	12/5
2^{-+}	${}^1D_2$	0	1
2^{-+}	${}^1D_2$	2	0
2^{++}	${}^3F_2$	0	49/15
2^{++}	${}^3F_2$	2	125/18
3^{++}	${}^3F_3$	0	0
3^{++}	${}^3F_3$	2	10/9
4^{++}	${}^3F_4$	0	0
4^{++}	${}^3F_4$	2	10/9

The spin-singlet quarkonia, i.e. $S = 0$, come with even J only and with $\Lambda = 0$. So the states with 0^{-+}, 2^{-+}, 4^{-+} cannot couple to $\Lambda = 2$. The same is true, obviously, for the scalar states 0^{++} (${}^3P_0$), for it would be impossible to couple to $\Lambda = 2$. The opposite is true for the familiar tensor states 2^{++} (${}^3P_2$), where only the $\Lambda = 2$ transition is allowed. On the other hand, the 2^{++} (${}^3F_2$) states communicate with both $\Lambda = 0$ and $\Lambda = 2$ amplitudes, although the $\Lambda = 2$ partial width is still more than twice that for $\Lambda = 0$. In general, $\Lambda = 2$ amplitudes dominate over $\Lambda = 0$ for all J^{++} for $J > 0$, i.e. 2^{++}, 3^{++}, 4^{++}, 5^{++}, etc.

References

[1] M. Jacob and G. C. Wick, Ann. Phys. (USA) **7**, 404 (1959).

[2] S. U. Chung, 'Spin Formalisms,' CERN preprint 71-8 (1971).

[3] S. U. Chung and H. J. Willutzki, 'Hadron Spectroscopy and *B* Physics at RHIC,' BNL preprint (1990); S. U. Chung, 'Reflectivity Eigenstates for Central Production,' BNL-QGS-94-23.

[4] S. U. Chung, 'Double-Pomeron and Two-Photon Processes at RHIC,' BNL preprint BNL-QGS-02-91.

[5] S. U. Chung, 'Search of Exotic Mesons at LEP,' CERN Informal Report (2001).

[6] See, for example, a note written by R. N. Cahn, in 'Review of Particle Physics,' Particle Data Group, Eur. Phys. J. **C 15**, 1 (2000).

[7] V. M. Budnev *et al.*, Phys. Reports **15C**, 181 (1975).

[8] S. U. Chung, Phys. Rev. D **48**, 1225 (1993); F. von Hippel and C. Quigg, Phys. Rev. **5**, 624 (1972).

[9] S. U. Chung, Phys. Rev. D **57**, 431 (1998).

[10] E. S. Ackleh, T. Barnes and F. E. Close, Phys. Rev. D **46**, 2257 (1992).