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General Formulation of Covariant Helicity-Coupling Amplitudes —An Explanatory Note II—

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Abstract

A general formulation has been given for constructing covariant helicity-coupling amplitudes involving two-body decays with arbitrary integer spins. In order to illustrate the principles, the case of a spin-1 object decaying into spin-2 and spin-1 particles is treated in some detail.

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1 Introduction

A general formulation has been given for constructing covariant helicity-coupling amplitudes involving two-body decays with arbitrary integer spins[1]. The purpose of this note is to give a more detailed account of the techniques developed in the previous paper[1]—by working out in some detail two examples of the J/ψ decay.

The main results of the paper[1] are first summarized in Sections 2 and 3. The rotational properties of canonical basis states are covered in Section 4, which are needed to obtain the main results but which have not been adequately emphasized in the original paper. In Sections 5 and 6 are given two examples of the J/ψ decay. Further developments of the J/ψ decay when one of its decay product is massless are given in Section 7. Finally, the concluding remarks are followed in Section 8.

It may be worth repeating here once more. The key principle which has been relied on is the so-called closure relationship

$$\sum_{jm} |jm\rangle \langle jm| = I \tag{1}$$

This identity in tensor formulation is termed the projection operator in the original paper[1] and again in this note. The reader will note that this principle has been used repeatedly in deriving the main result.

The reader may wish to consult additional works[2, 3, 4] for further examples of covariant helicity-coupling amplitudes.

2 Covariant Helicity-coupling Amplitudes

Consider a state with spin(parity)=$J(\eta_J)$ decaying into two states with $s(\eta_s)$ and $\sigma(\eta_\sigma)$. The decay amplitudes are given, in the rest frame of J,

$$\begin{aligned} \mathcal{M}^J_{\lambda\nu}(\vartheta,\varphi;M) &\propto \langle\vartheta,\varphi,\lambda\nu|JM\lambda\nu\rangle\langle JM\lambda\nu|\;\mathcal{M}\;|JM\rangle \\ &\propto \sqrt{2J+1}\,D^{J*}_{M\delta}(\varphi,\vartheta,0)\,F^J_{\lambda\nu} \end{aligned} \tag{2}$$

where $\mathcal{M}$ is the invariant operator for the decay, and λ and ν are the helicities of the two final-state particles 's' and 'σ' with $\delta = \lambda - \nu$. The symbol M stands for the z component of the spin J in a coordinate system fixed by production process. The helicities λ and ν are rotational invariants by definition. The direction of the break-up momentum of the decaying particle s is given by the angles ϑ and φ in the J rest frame. Let $\hat{x}$, $\hat{y}$ and $\hat{z}$ be the coordinate system fixed in the J rest frame. It is important to recognize, for applications to sequential decays, the exact nature of the body-fixed (helicity) coordinate system implied by the arguments of the D function given above. Let $\hat{x}_h$, $\hat{y}_h$ and $\hat{z}_h$ be the helicity coordinate system fixed by the s decay. Then by definition $\hat{z}_h$ describes the direction of the s in the J rest frame and the y axis is given by $\hat{y}_h \propto \hat{z} \times \hat{z}_h$ and $\hat{x}_h = \hat{y}_h \times \hat{z}_h$.

The helicity-coupling amplitude F^J given by

$$F^J_{\lambda\nu} \;\propto\; \langle JM\lambda\nu|\; \mathcal{M} \;|JM\rangle \tag{3}$$

is a rotational invariant and hence independent of M. Parity conservation in the decay leads to the relationship

$$F^J_{\lambda\nu} = \eta_{_J}\eta_s\eta_\sigma(-)^{J-s-\sigma}F^J_{-\lambda-\nu} \tag{4}$$

while, if the decay products 's' and 'σ' are identical, the following additional relationship holds

$$F^J_{\lambda\nu} = (-)^J F^J_{\nu\lambda} \tag{5}$$

In order to go over into the covariant description of the decay amplitudes, it is necessary to introduce further notations for the particles involved in the process. Let $p = q + k$, q and k be the four-momenta for the states 'J', 's' and 'σ' with masses W, m and μ

$$p^\alpha = (p_0, \boldsymbol{p}), \quad p^2 = W^2, \quad q^\alpha = (q_0, \boldsymbol{q}), \quad q^2 = m^2, \quad k^\alpha = (k_0, \boldsymbol{k}), \quad k^2 = \mu^2 \tag{6}$$

and let $r = q - k$ be the break-up four-momentum. Using the Lorentz metric $\bar{g}_{\alpha\beta}$, one has

$$p_\alpha = \bar{g}_{_{\alpha\beta}} p^\beta = (p_0, -\boldsymbol{p}\ ) \tag{7}$$

and similarly for the other four-vectors. In this paper, one has adopted the notations p, q, k and r to stand for *both* the four-momenta and the magnitudes of the 3-momenta. One can then define the following unitless quantities derived from them:

$$\gamma_s = \frac{q_0}{m}, \quad \gamma_s\beta_s = \frac{q}{m}, \quad \gamma_\sigma = \frac{k_0}{\mu} \quad \text{and} \quad \gamma_\sigma\beta_\sigma = \frac{k}{\mu} \tag{8}$$

Let $\phi^{J\,*}$, ω^s and ε^σ be the tensor wave functions for J, s and σ, respectively. They are defined exclusively along the z-axis, as spelled out in the next section. For ease of notation, the Lorentz indices for the tensors will be suppressed wherever possible. The invariant helicity-coupling amplitude can now be written

$$F^J_{\lambda\nu} \propto \sum_\ell \left[p^n, r^\ell, \omega^s(\lambda), \varepsilon^\sigma(-\nu), \phi^{J\,*}(\delta)\right] \tag{9}$$

where the square brackets imply that a Lorentz scalar is to be constructed by a suitable contraction of the indices and ℓ is the angular momentum to be defined more precisely later. There are two independent four vectors in the problem; one may use the pair (q, k) or equivalently $(p = q + k, r = q - k)$. If the parity is conserved in the decay, then ℓ is either odd or even but not both. The factor p^n for $n = 0$ or 1 completes the construction of a Lorentz scalar. So if there are odd number of Lorentz indices among r^ℓ, $\phi^{J\,*}$, ω^s and ε^σ, then p is necessary to form a scalar, and one must have $n = 1$. So for a parity conserving decay, one sees that $n = 0$ or $n = 1$ but not both.

The invariant helicity-coupling amplitude can be expressed as

$$F^J_{\lambda\nu} = \sum_{\ell S} g_{\ell\, S}\; A_{\ell S}(\lambda\nu) \tag{10}$$

where

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0\, S\delta|J\delta)(s\lambda\, \sigma{-}\nu|S\delta) \left(\frac{W}{W^0}\right)^n \left(\frac{r}{r_0}\right)^\ell f^s_\lambda(\gamma_s)\; f^\sigma_\nu(\gamma_\sigma) \tag{11}$$

where $n = 1$ for $s+\sigma+\ell-J$ odd and $n = 0$ otherwise and r_0 refers to the r corresponding to the nominal mass values W_0, m_0 and μ_0. The f-functions are normalized such that $f^j_m(\gamma) \to 1$ as $\gamma \to 1$. In this case, the formulas (10) and (11) reduce to the standard non-relativistic formula, given originally by Jacob and Wick[5] and treated again by Chung[6], in the limit $W \to W_0$, $r \to r_0$, $\gamma_s \to 1$ and $\gamma_\sigma \to 1$.

The functional dependence on the Lorentz factors are contained in the f-functions, given by

$$f^j_m(\gamma) = a^j(m) \sum_{m_0} b^j(m, m_0)\,(2\gamma)^{m_0} \tag{12}$$

where j, m and γ can stand for s, λ and γ_s or σ, ν and γ_σ, and m_0 ranges from 0(1), 2(3),$\cdots$, to $j-m$ =even(odd). The functions a^j and b^j are

$$\begin{aligned} a^j(m) &= \frac{(j+m)!(j-m)!}{(2j)!} \\ b^j(m, m_0) &= \frac{j!}{m_+!m_0!m_-!} \end{aligned} \tag{13}$$

A few examples of practical importance are given below. The function for $j = 1$ is given by

$$\begin{aligned} f^{(1)}_m(\gamma) &= 1 \quad \text{for} \quad m = \pm 1 \\ &= \gamma \quad \text{for} \quad m = 0 \end{aligned} \tag{14}$$

For j=2, the function f takes on the form

$$\begin{aligned} f^{(2)}_m(\gamma) &= 1 \quad \text{for} \quad m = \pm 2 \\ &= \gamma \quad \text{for} \quad m = \pm 1 \\ &= \frac{2}{3}\gamma^2 + \frac{1}{3} \quad \text{for} \quad m = 0 \end{aligned} \tag{15}$$

The function for $j = 3$ is given by

$$\begin{aligned} f^{(3)}_m(\gamma) &= 1 \quad \text{for} \quad m = \pm 3 \\ &= \gamma \quad \text{for} \quad m = \pm 2 \\ &= \frac{4}{5}\gamma^2 + \frac{1}{5} \quad \text{for} \quad m = \pm 1 \\ &= \frac{2}{5}\gamma^3 + \frac{3}{5}\gamma \quad \text{for} \quad m = 0 \end{aligned} \tag{16}$$

3 Wave Functions

The polarization four-vectors or wave functions appropriate for the particles $J = 1$, '$s = 1$' and '$\sigma = 1$' are well known. The relevant momenta are

$$\begin{array}{rcl} p^\alpha &=& (\quad W;\ 0,\ 0,\ 0\ ) \\ q^\alpha &=& (\quad q_0;\ 0,\ 0,\ q\ ) \\ &=& (\quad \gamma_s m;\ 0,\ 0,\ \gamma_s\beta_s m\ ) \\ k^\alpha &=& (\quad k_0;\ 0,\ 0,\ -q\ ) \\ &=& (\quad \gamma_\sigma \mu;\ 0,\ 0,\ -\gamma_\sigma\beta_\sigma\mu\ ) \\ r^\alpha &=& (\ q_0 - k_0;\ 0,\ 0,\ 2q\ ) \end{array} \tag{17}$$

where $W = q_0 + k_0$, $q_0 = \sqrt{m^2 + q^2}$, $k_0 = \sqrt{\mu^2 + q^2}$ and $r = q - k$. Let ϕ, ω and ε be the spin-1 wave function for J, s and σ. They are given by, in the J rest frame,

$$\begin{array}{rcll} \phi^\alpha(\pm) &=& \mp\frac{1}{\sqrt{2}} & (\quad 0;\ 1,\ \pm i,\ 0\ ) \\ \phi^\alpha(0) &=& & (\quad 0;\ 0,\ 0,\ 1\ ) \\ \omega^\alpha(\pm) &=& \mp\frac{1}{\sqrt{2}} & (\quad 0;\ 1,\ \pm i,\ 0\ ) \\ \omega^\alpha(0) &=& & (\quad \gamma_s\beta_s;\ 0,\ 0,\ \gamma_s\ ) \\ \varepsilon^\alpha(\pm) &=& \mp\frac{1}{\sqrt{2}} & (\quad 0;\ 1,\ \pm i,\ 0\ ) \\ \varepsilon^\alpha(0) &=& & (\ -\gamma_\sigma\beta_\sigma;\ 0,\ 0,\ \gamma_\sigma\ ) \end{array} \tag{18}$$

Note that

$$p_\alpha \phi^\alpha(\lambda) = q_\alpha \omega^\alpha(\lambda) = k_\alpha \varepsilon^\alpha(\lambda) = 0$$

for any λ.

These polarization four-vectors satisfy

$$\begin{array}{rcl} p_\alpha \phi^\alpha(m) &=& 0 \\ \phi^*_\alpha(m)\phi^\alpha(m') &=& -\delta_{mm'} \\ \sum_m \phi_\alpha(m)\phi^*_\beta(m) &=& \tilde{g}_{\alpha\beta}(W) \end{array} \tag{19}$$

where

$$\tilde{g}_{\alpha\beta}(W) = -\bar{g}_{\alpha\beta} + \frac{p_\alpha p_\beta}{W^2} \tag{20}$$

The last equation of (19) is the usual projection operator for spin-1 states. Note that, in the J rest frame, $\tilde{g}(W)$ has a zero time-component and $+1$ for the space-components. ω and ε satisfy similar conditions, but with their own $\tilde{g}$'s, i.e., $\tilde{g}(m)$ and $\tilde{g}(\mu)$.

The general spin-J wave function can be written

$$\phi^{\delta_1\cdots\delta_J}(m) = [a^J(m)]^{\frac{1}{2}} \sum_{m_0} 2^{m_0/2} \sum_P \phi^{\alpha_1}(+)\cdots\phi^{\beta_1}(0)\cdots\phi^{\gamma_1}(-)\cdots \tag{21}$$

where the indices $\{\delta_1\cdots\delta_J\}$ have been broken up into three distinct sets in the second summation, i.e., $\{\alpha_i\}$ with $(i = 1, m_+)$, $\{\beta_i\}$ with $(i = 1, m_0)$ and $\{\gamma_i\}$ with $(i = 1, m_-)$, where $m_\pm$ stands for the numbers of $\phi(\pm)$'s and m_0 for $\phi(0)$'s. Note that

$$J = m_+ + m_0 + m_- \quad \text{and} \quad m = m_+ - m_- \tag{22}$$

and that

$$2m_\pm = J \pm m - m_0 \tag{23}$$

It is apparent that the right-hand side must be always even. The first sum in (21) goes over the allowed values of m_0 given J and m. It is clear that the maximum is given by $J-m$, so that m_0 ranges from 0(1), 2(3),$\cdots$, to $J-m$ =even(odd). The second sum in (21) represents a summation on the permutations

$$\{(+)(+)\cdots(0)(0)\cdots(-)(-)\cdots\}$$

It is seen readily that the number of terms in the summation is given by $b^J(m, m_0)$. Note the following useful relationship:

$$\phi(-m) = (-)^m \phi^*(m) \tag{24}$$

It is best to illustrate these formulas with examples for $J = 1$, 2 and 3. For $J = 1$ one finds that (21) reduces to identities for $\phi(+)$ and $\phi(0)$. For $J = 2$, one finds

$$\phi^{\alpha\beta}(+2) \;=\; \phi^\alpha(+)\,\phi^\beta(+)$$

$$
\begin{aligned}
\phi^{\alpha\beta}(+1) &= \frac{1}{\sqrt{2}}\Big[\phi^{\alpha}(+)\,\phi^{\beta}(0)+\phi^{\alpha}(0)\,\phi^{\beta}(+)\Big] \\
\phi^{\alpha\beta}(0) &= \frac{1}{\sqrt{6}}\Big[\phi^{\alpha}(+)\,\phi^{\beta}(-)+\phi^{\alpha}(-)\,\phi^{\beta}(+)\Big]+\sqrt{\frac{2}{3}}\phi^{\alpha}(0)\,\phi^{\beta}(0)
\end{aligned}
\tag{25}
$$

and, for $J=3$, the wave functions take on the form

$$
\begin{aligned}
\phi^{\alpha\beta\gamma}(+3) &= \phi^{\alpha}(+)\,\phi^{\beta}(+)\,\phi^{\gamma}(+) \\
\phi^{\alpha\beta\gamma}(+2) &= \frac{1}{\sqrt{3}}\Big[\phi^{\alpha}(+)\,\phi^{\beta}(+)\,\phi^{\gamma}(0)+\phi^{\alpha}(+)\,\phi^{\beta}(0)\,\phi^{\gamma}(+)+\phi^{\alpha}(0)\,\phi^{\beta}(+)\,\phi^{\gamma}(+)\Big] \\
\phi^{\alpha\beta\gamma}(+1) &= \frac{1}{\sqrt{15}}\Big[\phi^{\alpha}(+)\,\phi^{\beta}(+)\,\phi^{\gamma}(-)+\phi^{\alpha}(+)\,\phi^{\beta}(-)\,\phi^{\gamma}(+)+\phi^{\alpha}(-)\,\phi^{\beta}(+)\,\phi^{\gamma}(+)\Big] \\
&\quad +\frac{2}{\sqrt{15}}\Big[\phi^{\alpha}(+)\,\phi^{\beta}(0)\,\phi^{\gamma}(0)+\phi^{\alpha}(0)\,\phi^{\beta}(+)\,\phi^{\gamma}(0)+\phi^{\alpha}(0)\,\phi^{\beta}(0)\,\phi^{\gamma}(+)\Big] \\
\phi^{\alpha\beta\gamma}(0) &= \frac{1}{\sqrt{10}}\Big[\phi^{\alpha}(0)\,\phi^{\beta}(+)\,\phi^{\gamma}(-)+\phi^{\alpha}(0)\,\phi^{\beta}(-)\,\phi^{\gamma}(+)+\phi^{\alpha}(-)\,\phi^{\beta}(+)\,\phi^{\gamma}(0) \\
&\quad +\phi^{\alpha}(+)\,\phi^{\beta}(-)\,\phi^{\gamma}(0)+\phi^{\alpha}(+)\,\phi^{\beta}(0)\,\phi^{\gamma}(-)+\phi^{\alpha}(-)\,\phi^{\beta}(0)\,\phi^{\gamma}(+)\Big] \\
&\quad +\sqrt{\frac{2}{5}}\phi^{\alpha}(0)\,\phi^{\beta}(0)\,\phi^{\gamma}(0)
\end{aligned}
\tag{26}
$$

A summary of notations used in this paper is given in Table I.

Table I. Two-body decay: $J \rightarrow s+\sigma$

	Parent	Daughter 1	Daughter 2
Spin	J	s	σ
Parity	η_J	η_s	η_σ
Helicity		λ	ν
Momentum	p	q	k
Energy	p_0	q_0	k_0
Mass	W	m	μ
Energy/Mass		γ_s	γ_σ
Velocity		β_s	β_σ
Wave function	$\phi^*(\lambda-\nu)$	$\omega(\lambda)$	$\varepsilon(-\nu)$

4 Rotational Properties

Let R be an arbitrary rotation defined in the J rest frame, i.e.

$$R\,\phi^\alpha(m) = \sum_{m'} \phi^\alpha(m')\, D^{(1)}_{m'm}(R) \tag{27}$$

Descriptions for ω and ε require boost operators. For the purpose, let $L_z(q)$ be the boost operator which takes a zero momentum in the J rest frame to q along the z-axis. Then, one has

$$\begin{aligned} \omega^\alpha(m) &= L_z(q)\,\phi^\alpha(m) \equiv \phi^\alpha(q\hat{z}, m) \\ \varepsilon^\alpha(m) &= L_z(-q)\,\phi^\alpha(m) \equiv \phi^\alpha(-q\hat{z}, m) \end{aligned} \tag{28}$$

Let $L(\vec{q})$ be a boost along the direction $\vec{q}$ and let $\mathring{R}$ be the roation which takes the z-axis to $\vec{q}$, i.e. $\vec{q} = q\mathring{R}\hat{z}$. Then one may write

$$L(\vec{q}) = \mathring{R}\, L_z(q)\, \mathring{R}^{-1} \tag{29}$$

From this, one sees[6] that

$$\begin{aligned} R\,\omega^\alpha(m) &= \sum_{m'} \phi^\alpha(\vec{q}, m')\, D^{(1)}_{m'm}(R) \\ R\,\varepsilon^\alpha(m) &= \sum_{m'} \phi^\alpha(-\vec{q}, m')\, D^{(1)}_{m'm}(R) \end{aligned} \tag{30}$$

where $\phi^\alpha(\vec{q}, m)$ is same as the $\phi^\alpha(m)$ with its z-axis fixed along $\vec{q}$.

The important conclusion here is that the boosted wave functions ω and ε act like a spin-1 wave functions under rotation in the J rest frame. The reader may note that ω and ε have been treated as canonical states[6]. In practice this does not matter since the wave functions we are dealing with are all aligned along the z-axis, where the difference between the helicity and canonical states disappears. However, it must be emphasized that the formulas derived in this note and in the paper of Ref[1] are based on the boosted wave functions being canonical.

5 $J/\psi \to a_2(1320)\rho$

As a concrete example, one treats here a case in which $J = 1$, $s = 2$ and $\sigma = 1$, i.e.

$$\phi^*(\delta) \to \omega^{(2)}(\lambda) + \sigma(-\nu) \tag{31}$$

All three wave functions are aligned along the z-axis and thus $\delta = \lambda - \nu$. This decay involves $S = 1$, 2 and 3 with ℓ taking on the even values 0, 2 or 4. Because of the property (30), the spin-2 $\omega^{(2)}(m)$ can be written

$$\omega^{(2);\alpha\beta}(\lambda) = \sum_{m_1\, m_2} (1m_1 1m_2|2\lambda)\, \omega^\alpha(m_1)\omega^\beta(m_2) \tag{32}$$

where $\omega(m)$ is that given in (18).

The key step to analyzing the decay process is to conjecture fictitious spin-1 wave functions ψ^α and ζ^α at rest in the parent rest frame, and they are used to project out the final-state wave functions $\omega^{(2)}$ and σ. Once again, these wave functions are aligned along the z-axis, so that they are identical in form to ϕ^α in (18). One builds up a spin-2 wave function $\psi^{\alpha\beta}$ out of ψ^α as follows:

$$\psi^{(2);\alpha\beta}(m_a) = \sum_{m_1\, m_2} (1m_1 1m_2|2m_a)\, \psi^\alpha(m_1)\psi^\beta(m_2) \tag{33}$$

It can shown that the formula above leads to (25), as expected. The projection operator corresponding to this fictitious state is

$$P^{(2);\alpha\beta;\alpha'\beta'} = \sum_{m_a} \psi^{(2);\alpha\beta}(m_a)\, [\psi^{(2);\alpha'\beta'}(m_a)]^* \tag{34}$$

It is necessary now to evaluate the contraction of the last factor above with $\omega^{(2)}$

$$[\psi^{(2)*}(m_a) \otimes \omega^{(2)}(\lambda)] \equiv [\psi^{(2);\alpha'\beta'}(m_a)]^*\, \tilde g_{\alpha'\gamma}(W)\, \tilde g_{\beta'\delta}(W)\, \omega^{(2);\gamma\delta}(\lambda) = f^{(2)}_\lambda(\gamma_s)\, \delta_{m_a\,\lambda} \tag{35}$$

where

$$f^{(2)}_\lambda(\gamma_s) = \sum_{m_1\, m_2} (1m_1 1m_2|2\lambda)^2\, \xi_s(m_1)\, \xi_s(m_2) \tag{36}$$

and

$$\begin{aligned} \xi_s(m) &= 1 \qquad \text{for} \qquad m = \pm 1 \\ &= \gamma_s \qquad \text{for} \qquad m = 0 \end{aligned} \tag{37}$$

Note that the formula (36) leads to the expression identical to that given in (15). Next a projection operator for spin 1 is constructed from ζ^α

$$P^{(1);\alpha\alpha'} = \sum_{m_b} \zeta^\alpha(m_b)\, \zeta^{\alpha'\,*}(m_b) \tag{38}$$

The appropriate contraction is

$$\begin{aligned} [\zeta(m_b)\, \otimes\, \sigma(-\nu)] &= \zeta^{\alpha'\,*}(m_b)\, \tilde{g}_{\alpha'\gamma}(W) \sigma^\gamma(-\nu) \\ &= \xi_\sigma(\nu)\, \delta_{m_b\,-\nu} = f^{(1)}_\nu(\gamma_\sigma)\, \delta_{m_b\,-\nu} \end{aligned} \tag{39}$$

To summarize the results so far, one may write

$$\begin{aligned} [P^{(2)}\, \otimes\, \omega^{(2)}(\lambda)] &= \psi^{(2)}(\lambda)\, f^{(2)}_\lambda(\gamma_s) \\ [P^{(1)}\, \otimes\, \sigma(-\nu)] &= \zeta(-\nu)\, f^{(1)}_\nu(\gamma_\sigma) \end{aligned} \tag{40}$$

This shows a *crucial* result that the problem has now been reduced to a completely non-relativistic one, because the wave functions ϕ^*, ψ and ζ are all at rest in the J rest frame. Introduce next the total intrinsic spin S as follows:

$$\psi^{(2);\alpha\beta}(\lambda)\, \zeta^\gamma(-\nu) = \sum_S (2\lambda\, 1\, {-\nu}|S\delta)\, \chi^{S;\alpha\beta;\gamma}(\delta) \tag{41}$$

So χ^S is always a rank-3 tensor irrespective of S.

The orbital angular momentum is treated in the same manner. In troduce a fictitious wave function $\tau^\ell(m)$ which is a rank-ℓ tensor and an analogue of the ket state $\ell m\rangle$. The corresponding projection operator is

$$P^\ell = \sum_m \tau^\ell(m)\, \tau^{\ell\,*}(m) \tag{42}$$

where the ℓ Lorentz indices for τ and $2\,\ell$ indices for P have been suppressed above. The contraction with ℓ multiples of r gives

$$[P^\ell\, \otimes\, rrr\cdots] \propto \tau^\ell(0)\, r^\ell \tag{43}$$

where, again, ℓ Lorentz indices have been left out from both sides of the eqation above. The orbital angular momentum should be perpendicular to the relative momentum $\vec{r}$, and $\vec{r}$ is aligned along the z-axis, so that one must have $m = 0$ above. Let Ψ be the wave function with spin J with a given ℓ and S. Then one has

$$\tau^\ell(0)\,\chi^S(\delta) = \sum_J (\ell 0\, S\delta|J\delta)\,\Psi^J_{\ell S}(\delta) \tag{44}$$

Once again, the ℓ Lorentz indices, as well as the incides α, β and γ, have been suppressed from both sides of the equation above.

It is helpful to retrace the steps, starting with the helicity-coupling amplitude ($J = 1$)

$$F^J_{\lambda\nu} \propto \sum_\ell g'_\ell\,[\phi^*(\delta), P^\ell \otimes rrr\cdots, P^{(2)} \otimes \omega^{(2)}(\lambda), P^{(1)} \otimes \sigma(-\nu)] \tag{45}$$

where g'_ℓ is the coefficient of expansion in ℓ and the square brackets signify that a suitable Lorentz scalar are to be constructed out all the elements. It should be noted that, in order to form a Lorentz scalar as outlined, ℓ must remain even. One continues retracing the steps

$$\begin{aligned} F^J_{\lambda\nu} &\propto \sum_\ell g'_\ell\,[\phi^*(\delta), \tau^\ell(0), \psi^{(2)}(\lambda), \zeta(-\nu)]\,r^\ell\,f^{(2)}_\lambda(\gamma_s)\,f^{(1)}_\nu(\gamma_\sigma) \\ &\propto \sum_{\ell S} g'_{\ell S}\,(2\lambda\,1{-}\nu|S\delta)\,[\phi^*(\delta), \tau^\ell(0), \chi^S(\delta)]\,r^\ell\,f^{(2)}_\lambda(\gamma_s)\,f^{(1)}_\nu(\gamma_\sigma) \\ &\propto \sum_{\ell S} g'_{\ell S}\,(\ell 0\,S\delta|J\delta)\,(2\lambda\,1{-}\nu|S\delta)\,[\phi^*(\delta) \otimes \Psi^J_{\ell S}(\delta)], r^\ell\,f^{(2)}_\lambda(\gamma_s)\,f^{(1)}_\nu(\gamma_\sigma) \end{aligned} \tag{46}$$

Introduce a new coefficient, as follows:

$$g_{\ell S} \propto g'_{\ell S}\,[\phi^*(\delta) \otimes \Psi^J_{\ell S}(\delta)] \tag{47}$$

Note that the quantity in the square brackets represents a contraction over a single Lorentz index; it is a scalar and is merely a numerical constant, devoid of any physical significance. With the definition given by (10), one finds

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0\,S\delta|J\delta)(2\lambda\,1{-}\nu|S\delta)\left(\frac{r}{r_0}\right)^\ell\,f^{(2)}_\lambda(\gamma_s)\;f^{(1)}_\nu(\gamma_\sigma) \tag{48}$$

where the first factor has been introduced, so that the formula above is identical to that of Jacob and Wick[5] in the non-relativistic limit except for the factors r^ℓ.

It is intructive to examine the contraction of indices in the factor

$$B_{\ell S}(\lambda\nu) \equiv [\phi^*(\delta), \tau^\ell(0), \chi^S(\delta)] \tag{49}$$

given in (46). One sees that, since χ^S is a tensor of rank 3,

$$\begin{aligned} B_{01}(\lambda\nu) &= \chi^{(1)}_{\alpha\beta;\rho}(\delta)\ \tilde{g}^{\beta\rho}(W)\ \tilde{g}^{\alpha\gamma}(W)\ \phi^*_\gamma(\delta) \\ B_{2S}(\lambda\nu) &= \left[\tau^{(2)}(0) : \chi^S(\delta)\cdot\phi^*(\delta)\right] \\ B_{43}(\lambda\nu) &= \left[\chi^{(3)}(\delta) \therefore \tau^{(4)}(0)\cdot\phi^*(\delta)\right] \end{aligned} \tag{50}$$

where S can be 1, 2 or 3 and the first subscript of A stands for ℓ. We have used a shorthand notation by which a single dot represents contraction of a single pair of Lorentz indices with repsect to the modified metric $g^{\alpha\gamma}(W)$. Likewise, a double (triple) dot signifies contraction of two (three) pairs of Lorentz indices. It should be noted that the rank-3 tensor χ is symmetric and traceless in the first two indices but *not* with the third (note the semicolon indicating this distinction). Note that there are five distinct amplitudes in the problem.

6 $J/\psi \to a_2(1320)b_1(1235)$

This hypotheical decay is similar in many respects with the example given in the previous section, except that the negative parity of the ρ has been changed to a positive one because of the presence of the b_1. Therefore, in order to conserve parity in the decay, one must restrict the orbital angular momentum ℓ to odd values, i.e. $\ell = 1$ or 3. The helicity-coupling amplitude ($J = 1$) is

$$F^J_{\lambda\nu} \propto \sum_\ell g'_\ell \left[p, \phi^*(\delta), P^\ell \otimes rrr\cdots, P^{(2)} \otimes \omega^{(2)}(\lambda), P^{(1)} \otimes \sigma(-\nu)\right] \tag{51}$$

where g'_ℓ is the coefficient of expansion in ℓ. In order to form a Lorentz scalar as outlined, one must introduce the four-momentum p for the parent particle J. Since the wave functions which appear above are all space-like in the J rest frame (i.e. no time components), it is also evident that a non-zero Lorentz scalar could be formed only by contracting with the totally anti-symmetric tensor $\epsilon_{\alpha\beta\gamma\delta}$. Evaluating the scalar in the J rest frame, one finds that the amplitude given above must now be proportional to W.

To analyze this decay process further, it is helpful to examine the counterpart of the scalars (49) for this decay

$$B_{\ell S}(\lambda\nu) \equiv [p, \phi^*(\delta), \tau^\ell(0), \chi^S(\delta)] \tag{52}$$

Written out in detail, they are

$$\begin{aligned}
B_{11}(\lambda\nu) &= \left[p, \tau^{(1)}(0)\cdot\chi^{(1)}(\delta), \phi^*(\delta)\right]\\
B_{12}(\lambda\nu) &= \left[p, \tau^{(1)}(0)\cdot\chi^{(2)}(\delta), \phi^*(\delta)\right]\\
B_{32}(\lambda\nu) &= \left[p, \chi^{(2)}(\delta) : \tau^{(3)}(0), \phi^*(\delta)\right]\\
B_{33}(\lambda\nu) &= \left[p, \chi^{(3)}(\delta) : \tau^{(3)}(0), \phi^*(\delta)\right]
\end{aligned} \tag{53}$$

where p is the four-momentum of J/ψ and the square brackets here indicate contraction of 4 Lorentz indices with the totally antisymmetric tensor $\varepsilon_{\alpha\beta\gamma\delta}$. One must keep in mind, once more, that χ^S is a rank-3 tensor regardless of the value of S.

The amplitudes given by (48) must now be replaced by

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0\, S\delta|J\delta)(2\lambda\, 1{-}\nu|S\delta)\left(\frac{W}{W_0}\right)\left(\frac{r}{r_0}\right)^\ell f^{(2)}_\lambda(\gamma_s)\; f^{(1)}_\nu(\gamma_\sigma) \tag{54}$$

This decay problem depends on four $g_{\ell S}$, i.e. g_{11}, g_{12}, g_{32} and g_{33} corresponding to those given in (53).

7 $\pi_2(1670) \to f_2(1270) + \pi$

This decay was considered in a previous paper by Chung[2]. The equations (125) from this paper is reproduced here:

$$\begin{aligned}
F^{(2)}_2 &= g_0 - \frac{1}{3}g_2\left(\frac{r}{r_0}\right)^2 + \frac{2}{35}g_4\left(\frac{r}{r_0}\right)^4\\
F^{(2)}_1 &= g_0\,\gamma_s + \frac{1}{6}g_2\left(\frac{r}{r_0}\right)^2\gamma_s - \frac{8}{35}g_4\left(\frac{r}{r_0}\right)^4\gamma_s\\
F^{(2)}_0 &= g_0\left(\frac{2}{3}\gamma_s^2 + \frac{1}{3}\right) + \frac{1}{3}\,g_2\left(\frac{r}{r_0}\right)^2\left(\frac{4}{3}\gamma_s^2 - \frac{1}{3}\right) + \frac{12}{35}\,g_4\left(\frac{r}{r_0}\right)^4\left(\frac{17}{18}\gamma_s^2 + \frac{1}{18}\right)
\end{aligned} \tag{55}$$

where g_0, g_2 and g_4 are three arbitrary complex constants. Now compare this to the formulas (10) and (11), which yield

$$\begin{aligned}
\sqrt{2}F_2^{(2)} &= \sqrt{\frac{2}{5}}g_0 + \frac{2}{\sqrt{7}}g_2\left(\frac{r}{r_0}\right)^2 + \frac{1}{\sqrt{35}}g_4\left(\frac{r}{r_0}\right)^4 \\
\sqrt{2}F_1^{(2)} &= \left[\sqrt{\frac{2}{5}}g_0 - \frac{1}{\sqrt{7}}g_2\left(\frac{r}{r_0}\right)^2 - \frac{4}{\sqrt{35}}g_4\left(\frac{r}{r_0}\right)^4\right]\gamma_2 \\
F_0^{(2)} &= \left[\frac{1}{\sqrt{5}}g_0 - \sqrt{\frac{2}{7}}g_2\left(\frac{r}{r_0}\right)^2 + 3\sqrt{\frac{2}{35}}g_4\left(\frac{r}{r_0}\right)^4\right]\left(\frac{2}{3}\gamma_s^2 + \frac{1}{3}\right)
\end{aligned} \tag{56}$$

8 Massless Objects in the Decay Product

Consider the following radiative decay of J/ψ

$$J/\psi \to f_2(1270) + \gamma \tag{57}$$

This decay is very similar to that of $J/\psi \to a_2(1320) + \rho$ except that the helicity $\nu = 0$ is not allowed for γ. It is clear that the invariant ℓS amplitude is

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0\, S\delta|J\delta)(2\lambda\, 1{-}\nu|S\delta)\left(\frac{r}{r_0}\right)^\ell f_\lambda^{(2)}(\gamma_s)\, \delta_{\nu,\pm1} \tag{58}$$

I do not know how gravitons come into play in hadronic decay processes. Consider, instead, a hypothetical massless glueball G with $J^{PC} = 2^{++}$ produced in a J/ψ decay as follows:

$$J/\psi \to G + \omega(782) \tag{59}$$

Formally, this decay is very similar to the decay $J/\psi \to a_2(1320) + \rho$ in the sense that $J^{\eta_J} = 1^-$, $s^{\eta_s} = 2^+$ and $\sigma^{\eta_\sigma} = 1^-$. One can expand the amplitude again in $S = 1$, 2 or 3 and $\ell = 0$, 2 or 4, since this procedure is based on projection operators and so it is not dependent on the nature of the final-state particles.

Since G is massless, its helicity λ is restricted to ± 2. This means that it is not possible to construct a spin-2 wave function in the style of (32). The simplest way out is to take the product ω^α and ω^β. It is clear that the invariant ℓS amplitude can be written

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0\, S\delta|J\delta)(2\lambda\, 1{-}\nu|S\delta)\left(\frac{r}{r_0}\right)^\ell \delta_{\lambda,\pm2}\, f_\nu^{(1)}(\gamma_\sigma) \tag{60}$$

9 Conclusions

A general formula for covariant helicity-coupling amplitudes for the decay $J \to s + \sigma$, where J, s and σ are arbitrary integers, has been given previously[1]. The main results are summarized in Sections 2 and 3.

In Section 4, the rotational properties—in the J rest frame—of the wave functions for s and σ are given in the canonical basis, and the results are applied to deriving the invariant amplitudes. It must be emphasized that the concept of canonical basis states is a requirement only for $s \geq 2$ and/or $\sigma \geq 2$. In particular, the results of Ref[1] can be applied to the decays involving photons as well as any other spin-1 particles in the final states. Because their helicities are restricted to ± 1, the Lorentz factors, undefined for the photons, never enter into the formula. Although gravitons require a different treatment in general, it is shown that our formula can be applied to this case as well—provided one imposes the contraint $\lambda = \pm 2$.

In two carefully chosen examples in Sections 5 and 6, it is shown how one may expand the helicity-coupling amplitudes in terms of the orbital angular momentum ℓ and the total intrinsic spin S—consistent with the requirement of Lorentz invariance. Section 7 covers the case when one of the decay product is massless.

Finally, one must bear in mind—and this has been the whole motivation for recent work in this field—that the so-called Zemach formalism[7] is fundamentally non-relativistic and require a modification as outlined in Ref[1], for applications to high-mass resonances which involve relativistic breakup momenta in their decays.

Appendix A: An Alternative Formulation

Consider once more the example of Section 5. Now we want to bypass (32) and couple a 'raw' form of rank-2 tensor for ω, i.e.

$$\begin{aligned}
\omega^{(2);\alpha\beta}(\lambda) &= \omega^{\alpha}(\pm 1)\omega^{\beta}(\pm 1) \qquad \text{for} \qquad \lambda = \pm 2 \\
&= \frac{1}{\sqrt{2}}\left[\omega^{\alpha}(\pm 1)\omega^{\beta}(0) + \omega^{\alpha}(0)\omega^{\beta}(\pm 1)\right] \qquad \text{for} \qquad \lambda = \pm 1 \\
&= \omega^{\alpha}(0)\omega^{\beta}(0) \qquad \text{for} \qquad \lambda = 0
\end{aligned} \tag{61}$$

So this tensor is definitely not a pure spin-2 tensor. But this is immaterial, since it is being mated to a pure spin-2 projection operator in the J rest frame. One finds

$$[\psi^{(2)*}(m_a)\,\otimes\,\omega^{(2)}(\lambda)] \equiv [\psi^{(2);\alpha'\beta'}(m_a)]^*\,\tilde{g}_{\alpha'\gamma}(W)\,\tilde{g}_{\beta'\delta}(W)\,\omega^{(2);\gamma\delta}(\lambda) = h^{(2)}_{\lambda}(\gamma_s)\,\delta_{m_a\,\lambda} \tag{62}$$

where

$$\begin{aligned}
h^{(2)}_{\lambda}(\gamma_s) &= 1 \qquad \text{for} \qquad \lambda = \pm 2 \\
&= \gamma_s \qquad \text{for} \qquad \lambda = \pm 1 \\
&= \sqrt{\frac{2}{3}}\,\gamma_s^2 \qquad \text{for} \qquad \lambda = 0
\end{aligned} \tag{63}$$

Appendix B: Sequential Decays

The amplitudes for sequential decays require Breit-Wigner forms for the isobars, which have been omitted here for brevity. The Cases 1 and 2 deal with the decays resulting in four spin-zero particles in the final state. The Case 3 covers the decays into five spin-zero particles.

Case 1: $s \to 0+0$ and $\sigma \to 0+0$

An example for this case would be $\eta_c(1S) \to \rho\rho$. The overall amplitude is, assuming an arbitrary J,

$$
\begin{aligned}
\mathcal{M}^J(\vartheta,\varphi;M) &\propto \sqrt{2s+1}\,\sqrt{2\sigma+1}\sum_{\lambda\nu}\mathcal{M}^J_{\lambda\nu}(\vartheta,\varphi;M)\left(\frac{r_s}{r_s^0}\right)^s\left(\frac{r_\sigma}{r_\sigma^0}\right)^\sigma \\
&\qquad D^{s*}_{\lambda 0}(\phi_s,\theta_s,0)\,D^{\sigma*}_{\nu 0}(\phi_\sigma,\theta_\sigma,0) \\
&\propto \sqrt{2J+1}\,\sqrt{2s+1}\,\sqrt{2\sigma+1}\sum_{\lambda\nu}D^{J*}_{M\delta}(\varphi,\vartheta,0)\,F^J_{\lambda\nu}\left(\frac{r_s}{r_s^0}\right)^s\left(\frac{r_\sigma}{r_\sigma^0}\right)^\sigma \\
&\qquad D^{s*}_{\lambda 0}(\phi_s,\theta_s,0)\,D^{\sigma*}_{\nu 0}(\phi_\sigma,\theta_\sigma,0)
\end{aligned}
\tag{64}
$$

where r_s and r_σ are the usual relative momenta in the rest frames for s and σ, respectively. r_s^0 and r_σ^0 are constants fixed, e.g. at the isobar masses. The helicity-coupling amplitude $F^J_{\lambda\nu}$ is given in (10). The $g_{\ell S}$'s are the parameters to be fitted experimentally.

The angles (θ_s,ϕ_s) and $(\theta_\sigma,\phi_\sigma)$ are defined the rest frames of s and σ, respectively, obtained by pure time-like Lorentz transformations from the J rest frame. Let $(\hat{x},\hat{y},\hat{z})$ be the coordinate system defined in the J rest frame. The vector $\vec{r}$ is described by the angles (ϑ,φ) in this system. The coordinate systems $(\hat{x}_s,\hat{y}_s,\hat{z}_s)$ and $(\hat{x}_\sigma,\hat{y}_\sigma,\hat{z}_\sigma)$ for s and σ are defined such that $\hat{z}_s \propto \vec{r}$, $\hat{z}_\sigma \propto -\vec{r}$, $\hat{y}_s \propto \hat{z}\times\hat{z}_s \propto \hat{z}\times\vec{r}$ and $\hat{y}_\sigma \propto \hat{z}\times\hat{z}_\sigma \propto -\hat{z}\times\vec{r}$.

There is an important constraint for the decay into $\rho\rho$. Because of the Bose symmetriation, one is restricted[8] to $I+\ell+S =$ even, where I is the isotopic spin of the parent particle J.

Case 2: $s \to v+0$, $v \to 0+0$ and $\sigma = 0$

An example is $\eta_c(1S) \to a_2(1320)\pi$. The amplitude is, assuming an arbitrary J,

$$
\begin{aligned}
\mathcal{M}^J(\vartheta,\varphi;M) &\propto \sqrt{2s+1}\sum_\lambda \mathcal{M}^J_{\lambda 0}(\vartheta,\varphi;M)\, D^{s*}_{\lambda\alpha}(\phi_s,\theta_s,0)\, G^s_\alpha \\
&\qquad \left(\frac{r_v}{r_v^0}\right)^v D^{v*}_{\alpha 0}(\phi_v,\theta_v,0) \\
&\propto \sqrt{2J+1}\sqrt{2s+1}\sum_\lambda D^{J*}_{M\lambda}(\varphi,\vartheta,0)\, F^J_\lambda\, D^{s*}_{\lambda\alpha}(\phi_s,\theta_s,0)\, G^s_\alpha \\
&\qquad \left(\frac{r_v}{r_v^0}\right)^v D^{v*}_{\alpha 0}(\phi_v,\theta_v,0)
\end{aligned}
\tag{65}
$$

where $\sigma = \nu = 0$ and α is the helicity of the v. r_v is the usual relative momentum in the v rest frame, and r_v^0 is that fixed at the nominal v mass. The coordinate sytem for the s is as defined for case 1. The coordinate sytem $(\hat{x}_v,\hat{y}_v,\hat{z}_v)$ for the v is defined in the v rest frame with $\hat{z}_v \propto \vec{r}_s$ and $\hat{y}_v \propto \hat{z}_s \times \hat{z}_v \propto \hat{z}_s \times \vec{r}_s$. The angles (θ_v,ϕ_v) describe the direction of $\vec{r}_v$ in this coordinate sytem.

This decay entails two helicity-coupling amplitudes F^J_λ for $J \to s+0$ and G^s_α for $s \to v+0$. They are given by

$$
F^J_\lambda = \sum_\ell g_\ell\ A_\ell(\lambda) \tag{66}
$$

where

$$
A_\ell(\lambda) = \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0\, s\lambda|J\lambda) \left(\frac{W}{W^0}\right)^n \left(\frac{r}{r_0}\right)^\ell f^s_\lambda(\gamma_s) \tag{67}
$$

and

$$
G^s_\alpha = \sum_{\ell'} h_{\ell'}\ B_{\ell'}(\alpha) \tag{68}
$$

where

$$
B_{\ell'}(\alpha) = \left(\frac{2\ell'+1}{2s+1}\right)^{\frac{1}{2}} (\ell' 0\, v\alpha|s\alpha) \left(\frac{m}{m_s^0}\right)^{n'} \left(\frac{r_s}{r_s^0}\right)^{\ell'} f^v_\alpha(\gamma_v) \tag{69}
$$

Here $n=1$ if $\ell+s-J$ is odd and $n=0$ otherwise; $n'=1$ if $\ell'+v-s$ is odd and $n'=0$ otherwise; m (m_s^0) is the effective (nominal) mass for the state s; and $\gamma_v = E_v/m_v$, where E_v and m_v are the energy and the effective mass of the v in the s rest frame. The constants g_ℓ and $h_{\ell'}$ are the parameters to be determined experimentally.

Case 3: $s \to v + 0$, $v \to 0 + 0$ and $\sigma \to 0 + 0$

Consider, for example, the decay $J/\psi \to a_2(1320)\,\rho$ with $a_2(1320) \to \rho\pi$ and $\rho \to \pi\pi$. The amplitude is, assuming an arbitrary J,

$$\begin{aligned}
\mathcal{M}^J(\vartheta,\varphi;M) &\propto \sqrt{2s+1}\,\sqrt{2\sigma+1}\,\sum_{\lambda\nu} \mathcal{M}^J_{\lambda\nu}(\vartheta,\varphi;M)\, D^{s*}_{\lambda\,\alpha}(\phi_s,\theta_s,0)\, G^s_\alpha \\
&\qquad \left(\frac{r_v}{r_v^0}\right)^v D^{v*}_{\alpha\,0}(\phi_v,\theta_v,0) \left(\frac{r_\sigma}{r_\sigma^0}\right)^\sigma D^{\sigma*}_{\nu\,0}(\phi_\sigma,\theta_\sigma,0) \\
&\propto \sqrt{2J+1}\,\sqrt{2s+1}\,\sqrt{2\sigma+1}\,\sum_{\lambda\nu} D^{J*}_{M\delta}(\varphi,\vartheta,0)\, F^J_{\lambda\nu}\, D^{s*}_{\lambda\,\alpha}(\phi_s,\theta_s,0)\, G^s_\alpha \\
&\qquad \left(\frac{r_v}{r_v^0}\right)^v D^{v*}_{\alpha\,0}(\phi_v,\theta_v,0) \left(\frac{r_\sigma}{r_\sigma^0}\right)^\sigma D^{\sigma*}_{\nu\,0}(\phi_\sigma,\theta_\sigma,0)
\end{aligned} \tag{70}$$

where $F^J_{\lambda\nu}$ is given in (10) and G^s_α is given in (68).

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