CERN INFORMAL REPORT

CERN-QGS-00-0801

Search for Exotic mesons at LEP
—Version VII—

S. U. Chung

EP Division, CERN, CH-1211 Genève 23, Switzerland

and

Physics Department, Brookhaven National Laboratory, Upton, NY 11973 *

December 19, 2003

abstract

The data from LEP experiments provide an opportunity to search for 4-quark exotic mesons, i.e. $(n\bar{n}+n\bar{n})$ where $n=\{u,d\}$, through two-photon fusion processes. The ultimate goal of this type of search is to establish existence of an exotic meson involving heavy quarks, e.g. $(c\bar{c}+n\bar{n})$.

* under contract number DE-AC02-98CH10886 with the U.S. Department of Energy

1 Introduction

At least two exotic mesons with $J^{PC} = 1^{-+}$ and masses at 1.4 and 1.6 GeV have been seen in experiments at BNL, CERN and IHEP/Protvino. The nature of these states are not yet determined—we only know that they must either be of the type ($n\bar{n}$ + gluon) or of the type ($n\bar{n} + n\bar{n}$) where $n = \{u, d\}$. If they happen to be produced in relative abundance in two-photon fusion processes, then they are likely to be 4-quark mesons, i.e. ($n\bar{n} + n\bar{n}$), as photons couple to charge and *not* to color.

As this note is intended for an audience not working on hadron spectroscopy, I start in Section 2 with a brief introduction to exotic mesons and their current status. Section 3 is devoted to a discussion of the quantum numbers a meson can acquire in the s-channel production in two-photon fusion processes. Brief concluding remarks are given in Section 4.

A study of double-Pomeron production as well as the two-photon fusion process was done in the context of a RHIC proposal[1]; the reader may consult the relevant preprints for details.

2 Exotic Mesons

The focus of hadron spectroscopy has now shifted away from the notion that a baryon is a three-quark system and a meson is a bound state of a quark and an antiquark. It has taken a very long time for the physics community to come to this conclusion, primarily because more complicated systems such as ($qqq + q\bar{q}$) or (qqq + gluon) are indistinguishable from a simple qqq state from their quantum numbers alone.

The situation improves in the case of mesons, since they can come in exotic J^{PC} combinations which lie outside of the realm of $q\bar{q}$. However, for a reason not entirely clear (at least to me), the exotic mesons are produced with cross sections at levels substantially lower than those of ordinary mesons. It may not be an overstatement to state that the quark model was born in the 60s and 70s precisely because of the small production cross-sections of the exotic mesons.

Be that as it may, let me first start out by stating a well known result. Let L and S be the orbital angular momentum and the total intrinsic spin of a $q\bar{q}$ system. Let s_1 and s_2 be the spins of q and $\bar{q}$, i.e. $s_1 = s_2 = 1/2$. Then, one readily finds

$$\begin{aligned} \vec{J} &= \vec{L} + \vec{S} \qquad \text{and} \qquad \vec{S} = \vec{s}_1 + \vec{s}_2 \\ P &= (-)^{L+1} \\ C &= (-)^{L+S} \\ PC &= (-)^{S+1} \end{aligned} \tag{1}$$

and the wave function in a state of J, L and S is given by

$$\begin{aligned} |JMLS\rangle = \sqrt{\frac{2L+1}{4\pi}} \sum_{\substack{m_1\, m_2 \\ m_s\, m}} &(s_1 m_1\, s_2 m_2 | S m_s)\, (S m_s\, L M_L | JM) \\ &\times \int d\Omega\, D^{L\,*}_{M_L\,0}(\phi, \theta, 0)\, |\Omega, s_1 m_1\, s_2 m_2\rangle \end{aligned} \tag{2}$$

where M is the z-component of spin J in a coordinate system given in the rest frame of $q\bar{q}$ and $\Omega = (\theta, \phi)$ specifies the direction of the breakup momentum in this rest frame. We use the notation $(j_1 m_1\, j_2 m_2 | jm)$ to denote the usual Clebsch-Gordan coefficients. The formula given here is of course elementary; I will show later how this can be generalized to include a gluonic meson: $q\bar{q}$ + gluon.

Given a set of relations (1), one finds that the following states are not allowed for a $q\bar{q}$ system, i.e

$$J^{PC} = 0^{--}, \quad 0^{+-}, \quad 1^{-+}, \quad 2^{+-}, \quad 3^{-+}, \ldots \tag{3}$$

In this note the exotic mesons listed above are designated using the notations given in Table I.

Table I: Notations for Exotic Mesons

$I^G(J^{PC})$	Exotics	$I^G(J^{PC})$	Exotics
$1^+(0^{--})$	ρ_0	$0^-(0^{--})$	ω_0
$1^+(0^{+-})$	b_0	$0^-(0^{+-})$	h_0
$1^-(1^{-+})$	π_1	$0^+(1^{-+})$	η_1
$1^+(2^{+-})$	b_2	$0^-(2^{+-})$	h_2
$1^-(3^{-+})$	π_3	$0^+(3^{-+})$	η_3

In the flux-tube model pioneered by Isgur and Paton[2] and further developed by others[3], an excited gluon in $q\bar{q}+$ gluon acquires a nonzero angular momentum component Λ along the $q\bar{q}$ axis. As a result, the P and the C eigenvalues decouple from the internal quantum numbers; only the product retains a connection, which in the lowest order vibrational modes of the string assume the form

$$\begin{gathered}\vec{J} = \vec{L} + \vec{S} \qquad \text{and} \qquad \vec{S} = \vec{s}_1 + \vec{s}_2 \\ PC = (-)^{S+N+1} = (-)^S, \quad N = 1\end{gathered} \tag{4}$$

where N is the 'photon number' which is proportional to the mass[4]. In the lowest-order phonon excitation, $N=1$. It is this change in the eigenvalue that allows for the existence of a J^{PC}-exotic gluonic hybrid. The corresponding transformation in the wave function is, suppressing the description for vibrational modes of a gluonic string[2],

$$\begin{aligned}|JMLS\,N\,\Lambda\rangle = \sqrt{\frac{2L+1}{4\pi}} \sum_{\substack{m_1\, m_2 \\ m_s\, m}} (s_1 m_1\, s_2 m_2 | S m_s)\,(S m_s\, L M_L | J M) \\ \times \int d\Omega\, D^{L\,*}_{M_L \Lambda}(\phi,\theta,0)\, |\Omega, s_1 m_1;\; s_2 m_2;\; N\,\Lambda\rangle\end{aligned} \tag{5}$$

where $D^J_{mm'}$ is the usual rotation functions, i.e. the standard representation of the rotation group. By definition, the quantum number Λ is a rotationally invariant quantity, as it is simply a 'body-fixed' z-component of the orbital angular momentum, i.e. it is the helicity of the excited flux along the $q\bar{q}$ axis. The lowest-order gluonic hybrid occurs evidently for $\Lambda = \pm 1$, and so one must have $L \geq 1$ in this case. If $S=1$, the allowed quantum numbers for the hybrid are, from (4), $J^{PC} = 0^{-+}/0^{+-}$, $1^{-+}/1^{+-}$ and $2^{-+}/2^{+-}$. If $S=0$, they are $1^{++}/1^{--}$. The slashes above emphasizes the PC-doubling which occur in the flux-tube model. The isovector and the isoscalar states for various types of quarks are predicted to have the masses as given in Table II.

Table II: Masses for Exotic Mesons

Constituents	Exotics	Mass (GeV)
$n\bar{n}+$ gluon	$b_0,\ h_0,\ \pi_1,\ \eta_1,\ b_2,\ h_2$	~ 1.9
$s\bar{s}+$ gluon	$b_{s0},\ h_{s0},\ \pi_{s1},\ \eta_{s1},\ b_{s2},\ h_{s2}$	~ 2.1
$c\bar{c}+$ gluon	$b_{c0},\ h_{c0},\ \pi_{c1},\ \eta_{c1},\ b_{c2},\ h_{c2}$	~ 4.3
$b\bar{b}+$ gluon	$b_{b0},\ h_{b0},\ \pi_{b1},\ \eta_{b1},\ b_{b2},\ h_{b2}$	~ 10.8

So far the BNL-E852, Crystal Barrel and VES/Protvino experiments have found candidates for two π_1's at masses 1.4 and 1.6 GeV[5]–[9]. A summary is given in Tables III and IV.

Table III: Parameters for $\pi_1(1400) \rightarrow \eta\pi$

Expt.	Mass(MeV)	Width(MeV)
KEK	1323.1 ± 4.6	143.2 ± 12.5
BNL('94)	$1370 \pm 16 {}^{+50}_{-30}$	$385 \pm 40 {}^{+65}_{-105}$
BNL('95)	$1359 {}^{+16}_{-14} {}^{+10}_{-24}$	$314 {}^{+31}_{-29} {}^{+9}_{-66}$
CB	$1400 \pm 20 \pm 20$	$310 \pm 50 {}^{+50}_{-30}$
CB	1360 ± 25	220 ± 90

Table IV: Parameters for $\pi_1(1600)$

Expt.	Mass(MeV)	Width(MeV)	Decay
BNL	$1593 \pm 8 {}^{+20}_{-47}$	$168 \pm 20 {}^{+150}_{-12}$	$\rho\pi$
BNL	1596 ± 8	387 ± 23	$\eta'\pi$
VES	1610 ± 20	290 ± 30	$\rho\pi$
			$\eta'\pi$
			$b_1\pi$

These candidates are presumably all of the type ($n\bar{n}$ + gluon) or ($n\bar{n} + n\bar{n}$). There may be in addition one more J^{PC}-exotic meson, $\pi_1(1900)$, coupling to $f_1(1285)\pi$, although the experimental evidence is not as solid as the two previous states. The question now is what are the nature of these exotic mesons. At some level, all must be a mixture of $(n\bar{n}+\text{gluon})$ and $(n\bar{n}+n\bar{n})$. But it is possible—and this is only a speculation at the moment—that the $\pi_1(1900)$ is largely gluonic, while the $\pi_1(1400)$ is mostly of a 4-quark constituency. The task is to search for these in a two-photon fusion process, which could provide an insight into the nature of these exotic mesons.

There are as yet no candidates corresponding to exotic strangeonia, i.e. $(s\bar{s}+\text{gluon})$ and/or $(s\bar{s}+n\bar{n})$, or—for that matter—exotic charmonia $(c\bar{c}+\text{gluon})$ and/or $(c\bar{c}+n\bar{n})$.

3 Two-Photon Fusion Processes

The idea is to use the LEP detectors to study the s-channel formation of J^{PC}-exotic mesons, in the processes (see Appendix A)

$$\begin{aligned} \gamma(t_1)+\gamma(t_2) &\to n\bar{n}+n\bar{n} \\ \gamma(t_1)+\gamma(t_2) &\to c\bar{c}+n\bar{n} \end{aligned} \tag{6}$$

where $\gamma(t)$ stands for the photon with a momentum transfer t. Let p_a and p_b be the lab momenta for the initial e^+ and e^-, respectively, and let p_a' and p_b' be those in the final state. Then, the momentum transfers are given by

$$\begin{aligned} t_1 &= (\,p_a-p_a'\,)^2 \\ t_2 &= (\,p_b-p_b'\,)^2 \end{aligned} \tag{7}$$

Assume for the purpose of discussion that $|t_1|<|t_2|$. For analysis of the states produced in the s-channel, the z-axis may be chosen such that

$$\vec{z} \;\propto\; (\,\vec{p}_a-\vec{p}_a{}'\,)-(\,\vec{p}_b-\vec{p}_b{}'\,) \tag{8}$$

Of course, if $|t_1|>|t_2|$, then the subscripts a and b should be interchanged. It is clear that with this choice of the axis one should not apply Bose symmetrization to the system containing two photons $\gamma(t_1)$ and $\gamma(t_2)$, for they are distinguishable by their momentum transfer.

A system of two identical particles, e.g. two off-shell photons at $t_1 = t_2$, must have[10]

$$L + S = \text{even} \tag{9}$$

where L is the orbital angular momentum in the rest frame and S is the total intrinsic spin, i.e. $S = 0$, 1 or 2. For an exotic meson under investigation, one must have $L = S = 1$ for $J^{PC} = 1^{-+}$, and $L = 3$ and $S = 1$ for $J^{PC} = 3^{-+}$. So, if Bose symmetrization applies, then exotic mesons are not possible for $S = 0$ or 2. To the extent that two photons $\gamma(t_1)$ and $\gamma(t_2)$ are distinguishable by their momentum transfer, all possible L and S are allowed. In particular, the $J^{PC} = 1^{-+}$ ($J^{PC} = 3^{-+}$) exotic meson is possible with $L = 1$ and $S = 0$ ($S = 2$).

Special relationships exist if a state decays to two identical massless photons. The decay amplitude may be written[10][11], in terms of the helicity-coupling amplitudes $F^a_{\lambda_1\,\lambda_2}$,

$$A \;\propto\; F^a_{\lambda_1\,\lambda_2}\, D^{J\,*}_{\Lambda\,\Lambda}(0,0,0) \tag{10}$$

where $\Lambda = \lambda_1 - \lambda_2$ and $a = J^P$. Note that $D^J_{\Lambda\,\Lambda}(0,0,0) = 1$ only if $J \geq |\Lambda|$. We have chosen a coordinate system in which the direction given by the two photons in their center-of-mass system is the z-axis and the azimuthal angle ϕ is fixed at zero. Note that the D^J must come with identical M components for it to be non-zero. Because of Bose symmetrization, one has[10][11]

$$F^a_{\lambda_1\,\lambda_2} = (-)^J\, F^a_{\lambda_2\,\lambda_1} \tag{11}$$

Since the decay particles are massless, the helicities cannot be zero. So, if $J =$ odd, then $F^a_{\pm\pm} = 0$ and the only allowed helicities are $\{\lambda_1, \lambda_2\} = \{\pm, \mp\}$ or $\Lambda = \pm 2$. But then the D-function is zero if $J = 1$, and one concludes that a spin-1 state cannot couple to two photons. There exists an additional constraint on the helicity-coupling amplitudes, coming from the parity conservation in the decay, i.e.

$$F^a_{\lambda_1\,\lambda_2} = \nu_1\,\nu_2\,\nu_a\; F^a_{-\lambda_1\,-\lambda_2} \tag{12}$$

where ν is the naturality of the particle, given by $\nu_1 = \eta_1(-)^{J_1}$, $\nu_2 = \eta_2(-)^{J_2}$ and $\nu_a = \eta_a(-)^J$. For photons one must have $\nu_1 = \nu_2 = +1$. If $J =$ odd, $\{\lambda_1, \lambda_2\} = \{\pm, \mp\}$ from (11), and (12) can be satisfied only if $\nu_a = (-)^J$ or $\eta_a = +1$. This means that no exotic states, for any J, can couple to on-shell $\gamma\gamma$.

The reason for a scrupulous choice of the coordinate system in the previous paragraph has been to avoid this problem: to repeat, a spin-1 state can couple to two photons, as long as they can be distinguished by their momentum transfer.

Now consider a case in which one or both photons are off-shell but neither is tagged. Then one has no way of distinguishing the two photons, and so the Bose symmetrization applies once again. The allowed helicities and spin-parities, derived from (11) and (12), are tabulated in Table Va.

Table Va: Allowed J^{PC}'s in two-photon Proceses
(Bose symmetrization imposed)

$\{\lambda_1, \lambda_2\}$	Λ	J^{PC}
$\{\pm, \pm\}$	0	even$^{\pm+}$, $J \geq 0$
$\{\pm, \mp\}$	± 2	J^{++}, $J \geq 2$
$\{0,\ 0\}$	0	even^{++}, $J \geq 0$
$\{0,\ \mp\}$	± 1	$J^{\pm+}$, $J \geq 1$
$\{\pm,\ 0\}$	± 1	$J^{\pm+}$, $J \geq 1$

A comment on the interference effects between different states listed above is in order. For the purpose, it is best to start with the distribution function $I(\tau)$, where $g(\tau)\, \mathrm{d}\tau$ is the relativistic phase-space element (see Appendix B). It may be expressed, in terms of the helicity-coupling amplitudes of (10),

$$I(\tau) = \sum_{\lambda_1, \lambda_2} \left| \sum_a F^a_{\lambda_1\, \lambda_2}(\tau) \right|^2 = \sum_{a\, a'} I_{a\, a'}(\tau) \tag{13}$$

where

$$I_{a\, a'}(\tau) = \sum_{\lambda_1, \lambda_2} F^a_{\lambda_1\, \lambda_2}(\tau)\, F^{a'\, *}_{\lambda_1\, \lambda_2}(\tau) \tag{14}$$

From (11), one sees that

$$
\begin{aligned}
I_{a\,a'}(\tau) &= \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_2\,\lambda_1}(\tau)\, F^{a'\,*}_{\lambda_2\,\lambda_1}(\tau) \\
&= (-)^{J-J'} \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_1\,\lambda_2}(\tau)\, F^{a'\,*}_{\lambda_1\,\lambda_2}(\tau) \\
&= (-)^{J-J'}\, I_{a\,a'}(\tau)
\end{aligned}
\tag{15}
$$

while, from (12), one finds

$$
\begin{aligned}
I_{a\,a'}(\tau) &= \sum_{\lambda_1,\lambda_2} F^{a}_{-\lambda_1\,-\lambda_2}(\tau)\, F^{a'\,*}_{-\lambda_1\,-\lambda_2}(\tau) \\
&= \nu_a\,\nu_{a'} \sum_{\lambda_1,\lambda_2} F^{a}_{\lambda_1\,\lambda_2}(\tau)\, F^{a'\,*}_{\lambda_1\,\lambda_2}(\tau) \\
&= \nu_a\,\nu_{a'}\, I_{a\,a'}(\tau)
\end{aligned}
\tag{16}
$$

So one concludes that the interference term should be zero, i.e. $I_{a\,a'} = 0\,(a \neq a')$, unless

$$
J - J' = \text{even} \qquad \underline{\text{and}} \qquad \nu_a = \nu_{a'} \tag{17}
$$

It is clear, in addition, that two states with different $\{\lambda_1, \lambda_2\}$ cannot interfere. There are in general nine elements in the set $\{\lambda_1, \lambda_2\}$, but because of the symmetry (11) and (12), there are only four independent elements; $\{+,\,+\}\,(\Lambda = 0)$, $\{+,\,-\}(\Lambda = 2)$, $\{0,\,0\}(\Lambda = 0)$ and $\{+,\,0\}(\Lambda = 1)$. There cannot be interference terms among the four elements listed above. So the partial-wave amplitudes $A_{\lambda_1\,\lambda_2}(J^{PC}\,\Lambda;\tau)$ are grouped together into new amplitudes $B^{(k\;\nu)}_{\lambda_1\,\lambda_2}(\Lambda;\tau)$, where the superscript k stands for either e (J=even) or o (J=odd) and ν denotes the naturality. Grouped into different Λ's, they are, for $\Lambda = 0$,

$$
\begin{aligned}
B^{(e+)}_{++}(0;\tau) &= A_{++}(0^{++}\,0;\tau) + A_{++}(2^{++}\,0;\tau) + A_{++}(4^{++}\,0;\tau) + \cdots \\
B^{(e-)}_{++}(0;\tau) &= A_{++}(0^{-+}\,0;\tau) + A_{++}(2^{-+}\,0;\tau) + A_{++}(4^{-+}\,0;\tau) + \cdots \\
B^{(e+)}_{0\,0}(0;\tau) &= A_{0\,0}(0^{++}\,0;\tau) + A_{0\,0}(2^{++}\,0;\tau) + A_{0\,0}(4^{++}\,0;\tau) + \cdots
\end{aligned}
\tag{18}
$$

for $\Lambda = 1$,

$$
\begin{aligned}
B^{(e+)}_{+\,0}(1;\tau) &= A_{+\,0}(2^{++}\,1;\tau) + A_{+\,0}(4^{++}\,1;\tau) + A_{+\,0}(6^{++}\,1;\tau) + \cdots \\
B^{(e-)}_{+\,0}(1;\tau) &= A_{+\,0}(2^{-+}\,1;\tau) + A_{+\,0}(4^{-+}\,1;\tau) + A_{+\,0}(6^{-+}\,1;\tau) + \cdots \\
B^{(o-)}_{+\,0}(1;\tau) &= A_{+\,0}(1^{++}\,1;\tau) + A_{+\,0}(3^{++}\,1;\tau) + A_{+\,0}(5^{++}\,1;\tau) + \cdots \\
B^{(o+)}_{+\,0}(1;\tau) &= A_{+\,0}(1^{-+}\,1;\tau) + A_{+\,0}(3^{-+}\,1;\tau) + A_{+\,0}(5^{-+}\,1;\tau) + \cdots
\end{aligned}
\tag{19}
$$

and, for $\Lambda = 2$,

$$
\begin{aligned}
B^{(e+)}_{+-}(2;\tau) &= A_{+-}(2^{++}\,2;\tau) + A_{+-}(4^{++}\,2;\tau) + A_{+-}(6^{++}\,2;\tau) + \cdots \\
B^{(o-)}_{+-}(2;\tau) &= A_{+-}(3^{++}\,2;\tau) + A_{+-}(5^{++}\,2;\tau) + A_{+-}(7^{++}\,2;\tau) + \cdots
\end{aligned}
\tag{20}
$$

Note that the states with $J^{PC} = 1^{-+}$, $J^{PC} = 3^{-+}$ and $J^{PC} = 5^{-+}$ are exotic and hence represent the mesons which cannot be quarkonia.

The overall distribution function is now given by a sum of the absolute squares of 9 non-interterfering amplitudes, as follows:

$$
\begin{aligned}
I(\tau) = {} & \left|B^{(e+)}_{++}(0;\tau)\right|^2 + \left|B^{(e-)}_{++}(0;\tau)\right|^2 + \left|B^{(e+)}_{0\,0}(0;\tau)\right|^2 \\
& + \left|B^{(e+)}_{+\,0}(1;\tau)\right|^2 + \left|B^{(e-)}_{+\,0}(1;\tau)\right|^2 + \left|B^{(o+)}_{+\,0}(1;\tau)\right|^2 + \left|B^{(o-)}_{+\,0}(1;\tau)\right|^2 \\
& + \left|B^{(e+)}_{+-}(2;\tau)\right|^2 + \left|B^{(o-)}_{+-}(2;\tau)\right|^2
\end{aligned}
\tag{21}
$$

If one assumes that both photons are on-shell (which may be a good approximation), then the distribution function is a sum of four terms only; the first two plus the last two terms in (21). Assume further that for a limited mass range, e.g. $\sqrt{s} < 3.0$ GeV, one can confine the partial waves to those with $J \leq 2$. In this case the last term of (21) disappears as well, since it consists of the states $|3^{++}\,2\rangle$ and possibly those with $J \geq 4$ which *do not interfere* with the first three terms. The distribution function can now be written

$$
\begin{aligned}
I(\tau) = {} & \left|A_{++}(0^{++}\,0;\tau) + A_{++}(2^{++}\,0;\tau)\right|^2 \\
& + \left|A_{++}(0^{-+}\,0;\tau) + A_{++}(2^{-+}\,0;\tau)\right|^2 + \left|A_{+-}(2^{++}\,2;\tau)\right|^2
\end{aligned}
\tag{22}
$$

Note the extreme simplicity of this formula; in particular, the partial wave $|2^{++}\,2\rangle$ is produced in isolation, i.e. with no interference with any other waves in the problem. However, note that the two states $|0^{++}\,0\rangle$ and $|2^{++}\,0\rangle$ are produced with 100% coherence between them, as are the two states $|0^{-+}\,0\rangle$ and $|2^{-+}\,0\rangle$.

Consider now a case in which one of the photons is tagged. It is clear that the photons can now be distinguished and so the Bose symmetrization does not apply. But the constraint (12) due to parity conservation must remain valid. The allowed J^{PC}'s for this case are listed in Table Vb.

Table Vb: Allowed J^{PC}'s in two-photon Proceses
(without Bose symmetrization)

$\{\lambda_1, \lambda_2\}$	Λ	J^{PC}
$\{\pm, \pm\}$	0	$J^{\pm +}$, $J \geq 0$
$\{\pm, \mp\}$	± 2	$J^{\pm +}$, $J \geq 2$
$\{0,\ 0\}$	0	even^{++}, odd^{-+}, $J \geq 0$
$\{0,\ \mp\}$	± 1	$J^{\pm +}$, $J \geq 1$
$\{\pm,\ 0\}$	± 1	$J^{\pm +}$, $J \geq 1$

There should be five independent elements in the set $\{\lambda_1, \lambda_2\}$; $\{+,\,+\}\,(\Lambda = 0)$, $\{+,\,-\}(\Lambda = 2)$, $\{0,\,0\}(\Lambda = 0)$, $\{0,\,-\}(\Lambda = 1)$ and $\{+,\,0\}(\Lambda = 1)$. If the untagged photon is very nearly on-shell, e.g. $\lambda_1 = 0$ is absent, then there would be just three independent elements, consisting of the first, second and the last element on the list above.

The partial-wave amplitudes $A_{\lambda_1 \lambda_2}(J^{PC}\,\Lambda;\tau)$ can be grouped together into new amplitudes $B^{(\nu)}_{\lambda_1 \lambda_2}(\Lambda;\tau)$, where ν denotes the naturality. Grouped into different Λ's, they are, for $\Lambda = 0$,

$$
\begin{aligned}
B^{(+)}_{++}(0;\tau) &= A_{++}(0^{++}\,0;\tau) + A_{++}(1^{-+}\,0;\tau) + A_{++}(2^{++}\,0;\tau) + \cdots \\
B^{(-)}_{++}(0;\tau) &= A_{++}(0^{-+}\,0;\tau) + A_{++}(1^{++}\,0;\tau) + A_{++}(2^{-+}\,0;\tau) + \cdots \\
B^{(+)}_{0\,0}(0;\tau) &= A_{0\,0}(0^{++}\,0;\tau) + A_{0\,0}(1^{-+}\,0;\tau) + A_{0\,0}(2^{++}\,0;\tau) + \cdots
\end{aligned}
\tag{23}
$$

for $\Lambda = 1$,

$$
\begin{aligned}
B^{(+)}_{+0}(1;\tau) &= A_{+0}(1^{-+}\,1;\tau) + A_{+0}(2^{++}\,1;\tau) + A_{+0}(3^{-+}\,1;\tau) + \cdots \\
B^{(-)}_{+0}(1;\tau) &= A_{+0}(1^{++}\,1;\tau) + A_{+0}(2^{-+}\,1;\tau) + A_{+0}(3^{++}\,1;\tau) + \cdots \\
B^{(+)}_{0-}(1;\tau) &= A_{0-}(1^{-+}\,1;\tau) + A_{0-}(2^{++}\,1;\tau) + A_{0-}(3^{-+}\,1;\tau) + \cdots \\
B^{(-)}_{0-}(1;\tau) &= A_{0-}(1^{++}\,1;\tau) + A_{0-}(2^{-+}\,1;\tau) + A_{0-}(3^{++}\,1;\tau) + \cdots
\end{aligned}
\tag{24}
$$

and, for $\Lambda = 2$,

$$
\begin{aligned}
B^{(+)}_{+-}(2;\tau) &= A_{+-}(2^{++}\,2;\tau) + A_{+-}(3^{-+}\,2;\tau) + A_{+-}(4^{++}\,2;\tau) + \cdots \\
B^{(-)}_{+-}(2;\tau) &= A_{+-}(2^{-+}\,2;\tau) + A_{+-}(3^{++}\,2;\tau) + A_{+-}(4^{-+}\,2;\tau) + \cdots
\end{aligned}
\tag{25}
$$

The overall distribution function is now given by a sum of the absolute squares of 9 non-interterfering amplitudes, as follows:

$$\begin{aligned}I(\tau) = &\left|B^{(+)}_{++}(0;\tau)\right|^2 + \left|B^{(-)}_{++}(0;\tau)\right|^2 + \left|B^{(+)}_{0\,0}(0;\tau)\right|^2 \\ &+ \left|B^{(+)}_{+0}(1;\tau)\right|^2 + \left|B^{(-)}_{+0}(1;\tau)\right|^2 + \left|B^{(+)}_{0-}(1;\tau)\right|^2 + \left|B^{(-)}_{0-}(1;\tau)\right|^2 \\ &+ \left|B^{(+)}_{+-}(2;\tau)\right|^2 + \left|B^{(-)}_{+-}(2;\tau)\right|^2\end{aligned} \tag{26}$$

If the untagged photon is very nearly on-shell, i.e. $\lambda_1 = 0$ is absent, then the distribution function consists of 6 non-interterfering amplitudes, given by

$$\begin{aligned}I(\tau) = &\left|B^{(+)}_{++}(0;\tau)\right|^2 + \left|B^{(-)}_{++}(0;\tau)\right|^2 \\ &+ \left|B^{(+)}_{+0}(1;\tau)\right|^2 + \left|B^{(-)}_{+0}(1;\tau)\right|^2 \\ &+ \left|B^{(+)}_{+-}(2;\tau)\right|^2 + \left|B^{(-)}_{+-}(2;\tau)\right|^2\end{aligned} \tag{27}$$

Consider now an approximation in which *both* photons are on-shell, even though one of them has been 'tagged.' Then the distribution function consists of four terms only; the first two plus the last two terms in (27). Suppose, in addition, that spins greater than 2 can be neglected near threshold, e.g. $\sqrt{s} < 3.0$ GeV. The distribution function may be written

$$\begin{aligned}I(\tau) = &\left|A_{++}(0^{++}\,0;\tau) + A_{++}(1^{-+}\,0;\tau) + A_{++}(2^{++}\,0;\tau)\right|^2 \\ &+ \left|A_{++}(0^{-+}\,1;\tau) + A_{++}(1^{++}\,1;\tau) + A_{++}(2^{-+}\,1;\tau)\right|^2 \\ &+ \left|A_{+-}(2^{++}\,2;\tau)\right|^2 + \left|A_{+-}(2^{-+}\,2;\tau)\right|^2\end{aligned} \tag{28}$$

Two spin-1 states, $|1^{++}\,0\rangle$ and $|1^{-+}\,0\rangle$, appear in the formula above; the latter is an exotic state. Note, in particular, that the two spin-1 states are produced coherently with the spin-0 and spin-2 states of the same naturality.

4 Conclusions

There exist two 'confirmed' $J^{PC} = 1^{-+}$ exotic mesons at 1.4 and 1.6 GeV, with an additional third state at 1.9 GeV likely to be confirmed in the near future. The LEP detector systems provide an important opportunity for searching for them in two-photon fusion processes, and the observation of such states at LEP would help clarify the nature of these exotic objects.

An exotic charmonium state, $(c\bar{c}+n\bar{n})$, with $J^{PC} = 1^{-+}$ or $J^{PC} = 3^{-+}$ could be detected as well at LEP by its decay mode $J/\psi + \phi$ (or $D^{**} + \bar{D} +$ charge-conjugate)[12], where the superscript $**$ has been used to denote any suitable excited state. The prime mass region to search for is 3.5–5.5 GeV for the exotic charmonium. According to P. Page[13], the 'hidden charm' decays, such as $J/\psi+\phi$, of a gluonic charmonium should be large—even dominant—if its mass falls below the $\bar{D}\,D^{**}$ threshold.

Tantalizing evidence for a state decaying into $J/\psi + \phi$ has recently appeared in Physical Review Letters[14]. The CLEO Collaboration finds the decay, with a branching ratio of about 10^{-4},

$$B \rightarrow J/\psi + \phi + K \tag{29}$$

for both B^+ and B^0 decay modes. The Dalitz plot of $m^2(J/\psi\,\phi)$ vs. $m^2(\phi\,K)$ shows a cluster at around $m(J/\psi\,\phi) = 4.4$ GeV. Observation of such a state at LEP would go a long way in clarifying the nature of an exotic charmonium $(c\bar{c} + n\bar{n})$. Of course, in order to have a convincing case, one must accumulate enough statistics on $J/\psi + \phi$ to perform spin-parity analysis.

From the conversations I have had so far with LEP people, it appears that the only hope for accumulating enough statistics on the $J/\psi + \phi$ system is to combine all four LEP data. Even then, one may need to go beyond two-photon fusion processes and search for the $J/\psi + \phi$ system inclusively. One must bear in mind, however, that one may wind up with a washed-out polarization of the putative $J^{PC} = 1^{-+}$ state coupling to $J/\psi+\phi$. Another point one must keep in mind is that an enhancement observed in an inclusive $J/\psi + \phi$ final state can either be $(c\bar{c} + \text{gluon})$ and/or $(c\bar{c} + n\bar{n})$.

A summary of the possible decay modes of the exotic mesons and the final states in which they can be detected are given in Tables VI, VII and VIII. It should be noted that no attempt has been made to be exhaustive in listing various decay modes; the main purpose here has been to present the reader with a broad scope of possible decays.

Table VIa: Decay Modes for $J^{PC} = 1^{-+}$ Exotic Mesons ($n\bar{n} + n\bar{n}$?)

I^G	Exotic Meson	Primary Decay(s)	Secondary Decay(s)
1^-	$\pi_1^0(1400)$ [a]	$\eta\pi^0$	$\eta \to \pi^+\pi^0\pi^-$, $\pi^0 \to \gamma\gamma$
		$\rho^\pm\pi^\mp$	$\rho^\pm \to \pi^\pm\pi^0$, $\pi^0 \to \gamma\gamma$
		$b_1^\pm(1235)\pi^\mp$	$b_1^\pm(1235) \to \pi^\pm\omega$, $\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$b_1^\pm(1235) \to \pi^\pm\omega$, $\omega \to \pi^0\gamma$, $\pi^0 \to \gamma\gamma$
1^-	$\pi_1^0(1600)$	$\eta'\pi^0$	$\eta' \to \pi^+\pi^-\eta$, $\eta \to \pi^+\pi^0\pi^-$, $\pi^0 \to \gamma\gamma$
			$\eta' \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$
		$\rho^\pm\pi^\mp$	$\rho^\pm \to \pi^\pm\pi^0$, $\pi^0 \to \gamma\gamma$
		$b_1^\pm(1235)\pi^\mp$	$b_1^\pm(1235) \to \pi^\pm\omega$, $\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$b_1^\pm(1235) \to \pi^\pm\omega$, $\omega \to \pi^0\gamma$, $\pi^0 \to \gamma\gamma$
		$\omega\rho^0$	$\rho^0 \to \pi^+\pi^-$, $\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$\rho^0 \to \pi^+\pi^-$, $\omega \to \pi^0\gamma$, $\pi^0 \to \gamma\gamma$

[a] This may be a four-quark state belonging to the **10** and $\overline{\mathbf{10}}$ representations, in which case there would not be an isoscalar partner.

Table VIb: Decay Modes for $J^{PC} = 1^{-+}$ Exotic Mesons ($n\bar{n} + n\bar{n}$?)

I^G	Exotic Meson	Primary Decay(s)	Secondary Decay(s)
0^+	$\eta_1(1600?)$	$\eta'\eta$	$\eta' \to \pi^+\pi^-\eta$, $\eta \to \pi^+\pi^0\pi^-$, $\pi^0 \to \gamma\gamma$
			$\eta' \to \pi^+\pi^-\eta$, $\eta \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$
		$\omega\omega$ [a]	$\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$\omega \to \pi^0\gamma$, $\pi^0 \to \gamma\gamma$
		$a_1^0(1260)\pi^0$	$a_1^0(1260) \to \rho^\pm\pi^\mp$, $\rho^\pm \to \pi^\pm\pi^0$, $\pi^0 \to \gamma\gamma$
		$a_2^0(1320)\pi^0$	$a_2^0(1320) \to \rho^\pm\pi^\mp$, $\rho^\pm \to \pi^\pm\pi^0$, $\pi^0 \to \gamma\gamma$

[a] Parity and Bose symmetrization require $L = S = 1$, where L is the orbital angular momentum and S is the total intrinsic of $\omega\omega$, i.e. $S = 0,\ 1,\ 2$.

Table VIc: Decay Modes for $J^{PC} = 1^{-+}$ Exotic Mesons ($n\bar{n} + n\bar{n}$?)

I^G	Exotic Meson	Primary Decay(s)	Secondary Decay(s)
1^-	$\pi_1^0(1900)$	$f_1(1285)\pi^0$	$f_1(1285) \to \pi^+\pi^-\eta$ $\eta \to \pi^+\pi^0\pi^-$, $\pi^0 \to \gamma\gamma$
			$f_1(1285) \to \pi^+\pi^-\eta$ $\eta \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$
			$f_1(1285) \to K^+K^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$f_1(1285) \to K^\pm K_s^0\pi^\mp$, $K_s^0 \to \pi^+\pi^-$ $\pi^0 \to \gamma\gamma$
			$f_1(1285) \to \rho^0\gamma$, $\rho^0 \to \pi^+\pi^-$ $\pi^0 \to \gamma\gamma$
		$b_1^\pm(1235)\pi^\mp$	$b_1^\pm(1235) \to \pi^\pm\omega$, $\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
0^+	$\eta_1(1900)$?	$f_1(1285)\eta$	$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$\eta \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$

Table VII: Decay Modes for $J^{PC} = 1^{-+}$ Exotic Mesons ($n\bar{n} + s\bar{s}$?)

I^G	Exotic Meson	Primary Decay(s)	Secondary Decay(s)
1^-	$\pi^0_{1s}(1900?)$	$K^*(890)\bar{K}$	$K^*(890) \to K\pi$
		$K^*_2(1430)\bar{K}$	$K^*_2(1430) \to K\pi$
		$K^*(1410)\bar{K}$	$K^*(1410) \to K^*(890)\pi \to K\pi\pi$
		$K_1(1400)\bar{K}$	$K_1(1400) \to K^*(890)\pi \to K\pi\pi$
		$K_1(1270)\bar{K}$	$K_1(1270) \to K\rho \to K\pi\pi$
		$\phi\rho^0$	$\phi \to K^+K^-$, $\rho^0 \to \pi^+\pi^-$
0^+	$\eta_{1s}(1900?)$	$K^*(890)\bar{K}$	$K^*(890) \to K\pi$
		$K^*_2(1430)\bar{K}$	$K^*_2(1430) \to K\pi$
		$K^*(1410)\bar{K}$	$K^*(1410) \to K^*(890)\pi \to K\pi\pi$
		$K_1(1400)\bar{K}$	$K_1(1400) \to K^*(890)\pi \to K\pi\pi$
		$K_1(1270)\bar{K}$	$K_1(1270) \to K\rho \to K\pi\pi$
		$\phi\omega$	$\phi \to K^+K^-$, $\omega \to \pi^+\pi^-\pi^0$

Table VIII: Decay Modes for $J^{PC} = 1^{-+}$ Exotic Mesons ($n\bar{n} + c\bar{c}$)

I^G	Exotic Meson	Primary Decay(s)	Secondary Decay(s)
1^-	$\pi_{1c}^0(4400?)$	$J/\psi + \rho^0$?	$J/\psi \to \ell^+\ell^-$, $\rho^0 \to \pi^+\pi^-$
		$J/\psi + b_1^0(1235)$?	$J/\psi \to \ell^+\ell^-$, $b_1^0(1235) \to \pi^0\omega$ $\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$J/\psi \to \ell^+\ell^-$, $b_1^0(1235) \to \pi^0\omega$ $\omega \to \pi^0\gamma$, $\pi^0 \to \gamma\gamma$
0^+	$\eta_{1c}(4400?)$	$J/\psi + \phi$?	$J/\psi \to \ell^+\ell^-$, $\phi \to K^+K^-$
			$J/\psi \to \ell^+\ell^-$, $\phi \to K_S^0\bar{K}_L^0$ $K_S^0 \to \pi^+\pi^-$, $K_L^0 \to \pi^+\pi^-\pi^0$
		$J/\psi + \omega$?	$J/\psi \to \ell^+\ell^-$, $\omega \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$
			$J/\psi \to \ell^+\ell^-$, $\omega \to \pi^0\gamma$, $\pi^0 \to \gamma\gamma$

The upgraded D0 with Roman pots and the COMPASS provide an opportunity for a double-Pomeron production of exotic mesons. These represent an exciting, complementary avenue for the exotic-meson search involving gluonic excitations, i.e. ($n\bar{n}$ + gluon) and ($c\bar{c}$ + gluon). It is clear that neither the two-photon nor the double-Pomeron production is not sufficient by itself to illuminate on the nature of the exotic mesons, but a comparative study of *both* should provide a powerful tool for an ultimate understanding of the constituents of the exotic mesons.

Acknowledgement

I am indebted to A. Litke/UC(Santa Cruz) and D. Ryabchikov/IHEP(Protvino) for several helpful discussions.

5 Appendix A

In this section, we give a general overview of the phenomenological formalism[15] necessary for the treatment of the processes

$$\begin{aligned} &e^+(p_a) + e^-(p_a) \to e^+(p'_a) + e^-(p'_b) + \gamma_1(p_1) + \gamma_2(p_2) \\ &\gamma_1(p_1) + \gamma_2(p_2) \to X(p) \end{aligned} \tag{30}$$

where the appropriate 4-momenta are given in parentheses. We shall use a convention in which *both* the 4-momenta *and* the magnitude of the 3-momenta are given the same notation. Let E_a and E_b the beam energies of e^+ and e^-in the collider frame; then one has $E_a = E_b$ and $p_a = p_b$, and one finds $E_a \simeq p_a$ and $E_b \simeq p_b$ in the approximation $m_e \approx 0$. Emitted photon energies are denoted ω_1 and ω_2. The relevant quantities of interest are

$$\begin{aligned} p_a &= p'_a + p_1, \quad \omega_1 = E_a - E'_a, \quad t_1 = p_1^2 = \omega_1^2 - p_{1L}^2 - p_{1T}^2 \\ p_b &= p'_b + p_2, \quad \omega_2 = E_b - E'_b, \quad t_2 = p_2^2 = \omega_2^2 - p_{2L}^2 - p_{2T}^2 \\ S &= (p_a + p_b)^2 = 4E_a\,E_b \\ s &= W^2 = p^2 = E^2 - p_L^2 - p_T^2, \quad \mathbf{p}_T = \mathbf{p}_{1T} + \mathbf{p}_{2T} \\ &= (p_1 + p_2)^2 \\ &= t_1 + t_2 + 2(\omega_1\,\omega_2 + p_{1L}p_{2L} - \mathbf{p}_{1T}\cdot\mathbf{p}_{2T}) \end{aligned} \tag{31}$$

where the subscripts L indicate the longitudinal components along the positive or the negative z-axis (defined to be along the e^+ beam direction), and the subscripts T stand for the 2-dimensional vectors transverse to the beam. Note that

$$\mathbf{p}'_{aT} + \mathbf{p}_{1T} = 0, \qquad \mathbf{p}'_{bT} + \mathbf{p}_{2T} = 0, \qquad \mathbf{p}'_{aT} + \mathbf{p}'_{bT} + \mathbf{p}_T = 0$$

Several useful relationships[15] are derived here. Define ϵ's through the magnitude of the 3-momenta

$$\begin{aligned} p'_a &= (1 - \epsilon_1)p_a \\ p'_b &= (1 - \epsilon_2)p_b \end{aligned} \tag{32}$$

Evidently, one must have $0 < \epsilon_1 < 1$ and $0 < \epsilon_2 < 1$. In the approximation $m_e \approx 0$, one has

$$\begin{aligned} E'_a &\simeq (1-\epsilon_1)E_a \\ E'_b &\simeq (1-\epsilon_2)E_b \end{aligned} \tag{33}$$

so that $\epsilon_1 \simeq \omega_1/E_a$ and $\epsilon_2 \simeq \omega_2/E_b$. Note that ϵ's thus reflect the fractional energy loss due to photon emission. There is one instance in which the approximation $m_e \approx 0$ cannot be used; it occurs in the expressions for the momentum transfers. Since one has

$$\begin{aligned} E_a &\simeq p_a + \frac{m_e^2}{2p_a} \\ E'_a &\simeq p'_a + \frac{m_e^2}{2p'_a} = p_a\,(1-\epsilon_1) + \frac{m_e^2}{2p_a\,(1-\epsilon_1)} \end{aligned}$$

and similarly for E_b and E'_b, the momentum transfers are given by[a]

$$\begin{aligned} -t_1 &\simeq (-t_1)_{\min} + 4p_a\,p'_a\,\sin^2\left(\frac{\alpha_a}{2}\right) \\ &\simeq -2m_e^2 + 2(E_a\,E'_a - p_a\,p'_a) + p_a\,p'_a\,\alpha_a^2 \simeq \frac{(m_e\,\epsilon_1)^2}{1-\epsilon_1} + \frac{p_{1T}^2}{1-\epsilon_1} \\ -t_2 &\simeq (-t_2)_{\min} + 4p_b\,p'_b\,\sin^2\left(\frac{\alpha_b}{2}\right) \\ &\simeq -2m_e^2 + 2(E_b\,E'_b - p_b\,p'_b) + p_b\,p'_b\,\alpha_b^2 \simeq \frac{(m_e\,\epsilon_2)^2}{1-\epsilon_2} + \frac{p_{2T}^2}{1-\epsilon_2} \end{aligned} \tag{34}$$

The angles α_a and α_b are the scattering angles (small) in the collider frame. We have put[b] $p_{1T} \simeq p'_a\,\alpha_a$ and $p_{2T} \simeq p'_b\,\alpha_b$.

The effective mass W can be expressed as

$$\begin{aligned} W^2 &= (p_1+p_2)^2 = (\omega_1+\omega_2)^2 - (\mathbf{p}_a - \mathbf{p}'_a + \mathbf{p}_b - \mathbf{p}'_b)^2 \\ &= 4\omega_1\omega_2 + (\omega_1-\omega_2)^2 - (p'_{aL} - p'_{bL})^2 - (\mathbf{p}'_{aT} + \mathbf{p}'_{bT})^2 \\ &= 4\omega_1\omega_2 + (E'_a - E'_b)^2 - (p'_a)^2 - (p'_b)^2 \\ &\qquad + 2p'_a p'_b\,[\cos\alpha_a\,\cos\alpha_b - \sin\alpha_a\,\sin\alpha_b\,\cos(\vartheta_1-\vartheta_2)] \\ &\simeq 4\omega_1\omega_2 - 2p'_a p'_b\,[1 - \cos\alpha_a\,\cos\alpha_b + \sin\alpha_a\,\sin\alpha_b\,\cos(\vartheta_1-\vartheta_2)] \end{aligned} \tag{35}$$

where the angles ϑ_1 and ϑ_2 specify the direction of the vectors $\mathbf{p}_{1T}$ and $\mathbf{p}_{2T}$ in the transverse plane. The formula above shows that $W^2 \simeq 4\omega_1\omega_2$, if the angles α_a and α_b are sufficiently

[a] The reader will note that the factor $(1-\epsilon_i)^{-1}$ is missing in Ref.[15], but this is nearly equal to 1 in the approximation considered there. We retain this factor for the first term to conform to the formula given in Ref.[16].

[b] The factor $(1-\epsilon_i)^{-1}$ for the second term in (34) comes naturally from this definition.

small. We now wish to find a relationship between W and the ϵ's defined above. Noting that [see (30)]

$$E = E_a + E_b - E'_a - E'_b$$
$$p_L = +p'_{aL} - p'_{bL}$$

and

$$p'_{aL} = p'_a\sqrt{1-\alpha_a^2} \simeq (1-\epsilon_1)\,p_a - \frac{p_{1T}^2}{2(1-\epsilon_1)\,p_a}$$
$$p'_{bL} = p'_b\sqrt{1-\alpha_b^2} \simeq (1-\epsilon_2)\,p_a - \frac{p_{2T}^2}{2(1-\epsilon_2)\,p_a}$$

one finds

$$\begin{aligned} E &\simeq (\epsilon_1+\epsilon_2)p_a \\ p_L &\simeq (-\epsilon_1+\epsilon_2)p_a - \frac{p_{1T}^2}{2(1-\epsilon_1)\,p_a} + \frac{p_{2T}^2}{2(1-\epsilon_2)\,p_a} \end{aligned} \tag{36}$$

These relationships enable one to express the transverse mass of X as

$$\begin{aligned} W_T^2 = W^2 + p_T^2 &= (E+p_L)(E-p_L) \\ &\simeq S\,\epsilon_1\,\epsilon_2 - \left(\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}\right) p_{1T}^2 - \left(\frac{\epsilon_2-\epsilon_1}{1-\epsilon_2}\right) p_{2T}^2 \end{aligned} \tag{37}$$

neglecting higher-order terms in p_{1T}^2 and p_{2T}^2. Since one has

$$p_T^2 = p_{1T}^2 + p_{2T}^2 + 2\,p_{1T}\,p_{2T}\,\cos(\vartheta_1-\vartheta_2) \tag{38}$$

or

$$\xi = \kappa + v_1^2 + v_2^2 + 2\,v_1\,v_2\,\cos(\vartheta_1-\vartheta_2) \tag{39}$$

one sees that

$$\xi \simeq \epsilon_1\,\epsilon_2 - (\epsilon_1-\epsilon_2)\left(\frac{v_1^2}{1-\epsilon_1} - \frac{v_2^2}{1-\epsilon_2}\right) \tag{40}$$

where $\xi = W_T^2/S$, $\kappa = W^2/S$, $v_1^2 = p_{1T}^2/S$ and $v_2^2 = p_{2T}^2/S$. It can be shown that the formula above can be obtained directly from (35) by retaining up to second-order terms in α_a and α_b.

The rapidity is given in the usual way

$$\begin{aligned} y &= \tanh^{-1}\left(\frac{p_L}{E}\right) = \frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right) \\ &\simeq \frac{1}{2}\ln\left(\epsilon_2 - \frac{v_1^2}{1-\epsilon_1} + \frac{v_2^2}{1-\epsilon_2}\right) - \frac{1}{2}\ln\left(\epsilon_1 + \frac{v_1^2}{1-\epsilon_1} - \frac{v_2^2}{1-\epsilon_2}\right) \end{aligned} \tag{41}$$

It can be shown that

$$\begin{aligned}\epsilon_1 &\simeq \sqrt{\xi}\,\mathrm{e}^{-y} - \frac{v_1^2}{1-\epsilon_1} + \frac{v_2^2}{1-\epsilon_2}\\ \epsilon_2 &\simeq \sqrt{\xi}\,\mathrm{e}^{\,y} + \frac{v_1^2}{1-\epsilon_1} - \frac{v_2^2}{1-\epsilon_2}\end{aligned} \tag{42}$$

neglecting higher-order terms in v_1^2 and v_2^2. In order to proceed further, one needs to make an *additional* assumption that ϵ_1 and ϵ_2 are small compared to 1. Since v_1^2 and v_2^2 are already small, this restriction on ϵ_1 and ϵ_2 is not very strigent. We merely require that they be far from 1. Note that this is equivalent to a restriction on the size of the rapidity y; it implies that y cannot be arbitrarily large. Replacing $(1-\epsilon_i)^{-1}$ by $(1+\epsilon_i)$ and solving for ϵ_1 and ϵ_2, one finds the simple result

$$\begin{aligned}\epsilon_1 &\simeq \sqrt{\xi}\,\mathrm{e}^{-y}\,(1-v_1^2) + \sqrt{\xi}\,\mathrm{e}^{\,y}\,v_2^2 - v_1^2 + v_2^2\\ \epsilon_2 &\simeq \sqrt{\xi}\,\mathrm{e}^{\,y}\,(1-v_2^2) + \sqrt{\xi}\,\mathrm{e}^{-y}\,v_1^2 + v_1^2 - v_2^2\end{aligned} \tag{43}$$

It should be noted that these formulas give $\epsilon_1 + \epsilon_2 = \sqrt{\xi}\,(\mathrm{e}^{\,y} + \mathrm{e}^{-y})$, as do those of (42).

The evaluation of the invariant phase-space element for the process $e^+e^- \to e^+e^-X$ requires a factor

$$\mathrm{d}\Phi = \frac{\mathrm{d}^3\mathbf{p}'_a}{2E'_a}\frac{\mathrm{d}^3\mathbf{p}'_b}{2E'_b} = \frac{1}{4}\frac{\mathrm{d}p'_{aL}}{E'_a}\frac{\mathrm{d}p'_{bL}}{E'_b}\,\mathrm{d}^2\mathbf{p}_{1T}\,\mathrm{d}^2\mathbf{p}_{2T} \tag{44}$$

where all other revelant factors, including the δ-function expressing the four-momentum conservation, have been obsorbed into the formula for the two-photon cross section. From $\omega_1 = E_a - E'_a$ and $\omega_2 = E_a - E'_b$, one finds that

$$\begin{aligned}\mathrm{d}\Phi &= \frac{1}{4}\frac{\mathrm{d}\omega_1}{p'_{aL}}\frac{\mathrm{d}\omega_2}{p'_{bL}}\,\mathrm{d}^2\mathbf{p}_{1T}\,\mathrm{d}^2\mathbf{p}_{2T}\\ &\simeq \frac{1}{4}\left(\frac{\mathrm{d}\epsilon_1}{1-\epsilon_1}\right)\left(\frac{\mathrm{d}\epsilon_2}{1-\epsilon_2}\right)\mathrm{d}^2\mathbf{p}_{1T}\,\mathrm{d}^2\mathbf{p}_{2T}\\ &\simeq \frac{1}{16}\left(\frac{\mathrm{d}\epsilon_1}{1-\epsilon_1}\right)\left(\frac{\mathrm{d}\epsilon_2}{1-\epsilon_2}\right)\mathrm{d}p_{1T}^2\,\mathrm{d}p_{2T}^2\,\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2\\ &\simeq \frac{1}{16}\,\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2\,\mathrm{d}t_1\,\mathrm{d}t_2\,\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2\end{aligned} \tag{45}$$

For the application considered here, one needs to transform the phase-space variables into those involving the effective mass W for X. Observe that

$$\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2 \simeq (1-v_1^2-v_2^2)\,\mathrm{d}y\,\mathrm{d}\xi \tag{46}$$

evaluating, from (43), the appropriate Jacobian. One obtains[c]

$$
\begin{aligned}
\mathrm{d}\Phi &\simeq \left(\frac{t_1\,t_2}{16}\right)\,\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2\,\left(\frac{\mathrm{d}t_1}{t_1}\right)\,\left(\frac{\mathrm{d}t_2}{t_2}\right)\,\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2 \\
&\simeq \left(\frac{t_1\,t_2}{16}\right)\,(1-v_1^2-v_2^2)\,\mathrm{d}y\,\mathrm{d}\kappa\,\left[\frac{\mathrm{d}p_{1T}^2}{(m_e\,\epsilon_1)^2+p_{1T}^2}\right]\,\left[\frac{\mathrm{d}p_{2T}^2}{(m_e\,\epsilon_2)^2+p_{2T}^2}\right]\,\mathrm{d}\vartheta_1\,\mathrm{d}\vartheta_2
\end{aligned}
\tag{47}
$$

We do not integrate over these angles ϑ_1 and ϑ_2, because p_T^2 given by (38) enters into the definition of ξ and hence it must be held fixed [see (42)]. Note that the transition from $\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2 \to \mathrm{d}y\,\mathrm{d}\xi \to \mathrm{d}y\,\mathrm{d}\kappa$ in (47) can occur only when $\mathbf{p}_{1T}$ and $\mathbf{p}_{2T}$, and hence p_T^2, remain specified.

In the usual approximation[16], the cross section for $e^+e^- \to e^+e^-X$ is related to that for $\gamma\gamma \to X$ through

$$
\mathrm{d}\sigma(e^+e^- \to e^+e^-X; S) = \mathrm{d}N_1\,\mathrm{d}N_2\,\mathrm{d}\sigma(\gamma\gamma \to X; s) \tag{48}
$$

where N_1 and N_2 are the numbers of photons with energies ω_1 and ω_2 and momentum-transfers t_1 and t_2, respectively. They are given by

$$
\mathrm{d}N_i = F(t_i,\epsilon_i)\,\left(\frac{\mathrm{d}\vartheta_i}{2\pi}\right)\,\left(\frac{\mathrm{d}\epsilon_i}{\epsilon_i}\right)\,\left(\frac{\mathrm{d}t_i}{t_i}\right) \tag{49}
$$

where

$$
F(t_i,\epsilon_i) = \left(\frac{\alpha}{\pi}\right)\,\left[1-\epsilon_i+\left(\frac{1}{2}+\frac{m_e^2}{t_i}\right)\epsilon_i^2\right] \tag{50}
$$

for $i = 1$ or 2. We have restored the dependence on the angles ϑ_1 and ϑ_2 in the phase-space element, because the transverse mass W_T depends on p_T which in turn is a function of the angles [see (38)]. It is convenient to recast the formula

$$
\frac{\mathrm{d}N_i}{\mathrm{d}\epsilon_i\,\mathrm{d}u_i^2} = G(u_i,\epsilon_i)\,\left(\frac{1}{\epsilon_i^2+u_i^2}\right)\,\left(\frac{\mathrm{d}\vartheta_i}{2\pi}\right) \tag{51}
$$

where

$$
G(u_i,\epsilon_i) = \left(\frac{\alpha}{\pi}\right)\,\left[\frac{1-\epsilon_i}{\epsilon_i}+\frac{\epsilon_i}{2}-\frac{(1-\epsilon_i)\,\epsilon_i}{\epsilon_i^2+u_i^2}\right] \tag{52}
$$

and $u_i = p_{iT}/m_e$. Now the formula has been given entirely in unitless variables. This formula takes on a particularly simple form at $u_i = 0$. Noting that the dependence on ϑ_i disappears in this limit, one finds

$$
\left[\frac{\mathrm{d}N_i}{\mathrm{d}\epsilon_i\,\mathrm{d}u_i^2}\right]_{u_i=0} = \left(\frac{\alpha}{2\pi}\right)\,\frac{1}{\epsilon_i} \tag{53}
$$

[c] The reader will note that the phase-space formula given here can be obtained from a very different approach, as in Ref.[15]. It is seen that the factors $(1-\epsilon_1)^{-1}$ and $(1-\epsilon_2)^{-1}$ are missing in the formula in the cited reference. However, the difference is minor, since ϵ's are rather small in most applications.

As long as ϵ_i remains finite, the photon flux is nonzero in the limit $u_i = 0$.

The differential cross section can be cast into

$$\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^- X; S)}{\mathrm{d}\epsilon_1\,\mathrm{d}\epsilon_2\,\mathrm{d}\tau} \simeq \int F(t_1,\epsilon_1)\,F(t_2,\epsilon_2)\left(\frac{\mathrm{d}\vartheta_1}{2\pi}\right)\left(\frac{\mathrm{d}\vartheta_2}{2\pi}\right)\left(\frac{\mathrm{d}t_1}{\epsilon_1\,t_1}\right)\left(\frac{\mathrm{d}t_2}{\epsilon_2\,t_2}\right) \times \frac{\mathrm{d}\sigma(\gamma\gamma\to X; s)}{\mathrm{d}\tau} \tag{54}$$

where $g(\tau)\,\mathrm{d}\tau$ is the invariant phase-space element given by the final state which the X decays into (see Appendix B). Note that the integration can be readily performed using the formulas (34), (37), (42) and (38). Using (47), the formula can be rewritten[d]

$$\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^- X; S)}{\mathrm{d}y\,\mathrm{d}\kappa\,\mathrm{d}\tau} \simeq H_0(z_1,z_2,y,\kappa)\,\frac{\mathrm{d}\sigma(\gamma\gamma\to X; s)}{\mathrm{d}\tau} \tag{55}$$

where

$$H_0(z_1,z_2,y,\kappa) = \int (1-v_1^2-v_2^2)\,G(u_1,\epsilon_1)\,G(u_2,\epsilon_2)\left(\frac{\mathrm{d}u_1^2}{\epsilon_1^2+u_1^2}\right)\left(\frac{\mathrm{d}u_2^2}{\epsilon_2^2+u_2^2}\right)\left(\frac{\mathrm{d}\vartheta}{2\pi}\right) \tag{56}$$

and $z_i = u_{i\,\max} = p_{iT\,\max}/m_e$. The range of integration is $(0\to 2\pi)$ for ϑ and $(0\to z_i^2)$ for u_i^2, where $p_{1T\,\max} \simeq p_a'\,\alpha_{\max}$ and $p_{2T\,\max} \simeq p_b'\,\alpha_{\max}$. ($\alpha_{\max}$ is the maximum for α_a, fixed by the collider and detector geometry.)

Most of the literature on two-photon processes deal with the case in which $W \gg p_T$, leading to a simple closed form

$$\frac{\mathrm{d}\sigma(e^+e^- \to e^+e^- X; S)}{\mathrm{d}y\,\mathrm{d}\kappa\,\mathrm{d}\tau} \simeq H(z_1,\epsilon_1)\,H(z_2,\epsilon_2)\,\frac{\mathrm{d}\sigma(\gamma\gamma\to X; s)}{\mathrm{d}\tau} \tag{57}$$

where

$$H(z_i,\epsilon_i) = \left(\frac{\alpha}{\pi}\right)\left[\left(\frac{1-\epsilon_i}{\epsilon_i}+\frac{\epsilon_i}{2}\right)\ln\left(1+z_i^2/\epsilon_i^2\right) - \frac{1-\epsilon_i}{\epsilon_i}\left(1-\frac{1}{1+z_i^2/\epsilon_i^2}\right)\right] \tag{58}$$

However, in an application in which a study is made of the mesons in the mass range comparable to p_T, one has to resort to the formula (55). This is illustrated in Figs.1 and 2.

Appendix B

We consider here production of a given J^{PC} state in the s-channel in the reaction

$$\gamma\gamma \to \rho_1^0\rho_2^0 \to \pi_1^+\pi_2^-\pi_3^+\pi_4^- \tag{59}$$

[d] We can set $\vartheta_2 = 0$ and then integrate over ϑ_2 in (38), so that p_T^2 is a function of $\vartheta \equiv \vartheta_1$ only. Note that the remaining integration is over three variables: ϑ, $u_1^2 = (p_{1T}/m_e)^2$ and $u_2^2 = (p_{2T}/m_e)^2$.

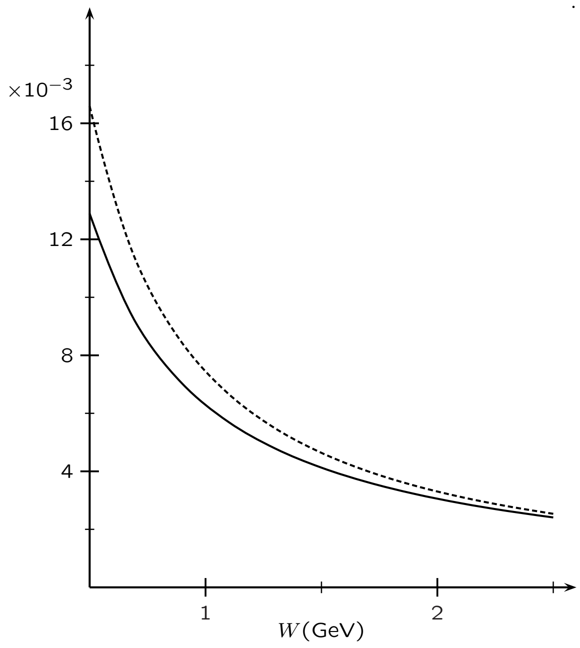

Figure 1: Two-photon flux for $y = 0$ with the mass W in the range $0.5 < W < 2.5$ GeV. The solid curve corresponds to $H_0(z_1, z_2, y, \kappa)$ [see (56)] and the dashed curve to $H(z_1, \epsilon_1)\, H(z_2, \epsilon_2)$ [see (58)]. We have used $\sqrt{S} = 175$ GeV and $\alpha_{\max} = 30$ mrad.

where $\rho_1^0 \to \pi_1^+\pi_2^-$ and $\rho_2^0 \to \pi_3^+\pi_4^-$. Note that $C = +1$ for the 4π system since it results from the two-photon initial system. Since the 4π system must have $G = +1$, it should have $I = 0$ or $I = 2$. So, most likely, the 4π system comes the quantum numbers $I^G\,(J^{PC}) = 0^+\,(J^{P\,+})$, including the J^{PC} exotic mesons which could be produced in the s-channel in (59).

The relativistic phase-space element $g(\tau)\,\mathrm{d}\tau$, appropriate for reaction (59),

$$\begin{aligned} g(\tau) &= \frac{p_\rho}{W}\, k_1\, k_2 \\ \mathrm{d}\tau &= \mathrm{d}w_1\, \mathrm{d}w_2\, \mathrm{d}\!\cos\theta\, \mathrm{d}\Omega_1\, \mathrm{d}\Omega_2 \end{aligned} \tag{60}$$

where $W = \sqrt{s}$ is the effective mass of the $\rho^0\rho^0$ system, equal to the center-of-mass (CM) for the reaction (59); p_ρ is the CM breakup momentum of the $\rho^0\rho^0$ system; $w_1(w_2)$ is the effective mass of the $\pi_1^+\pi_2^-(\pi_3^+\pi_4^-)$; $k_1(k_2)$ is the breakup momentum of the $\pi_1^+\pi_2^-(\pi_3^+\pi_4^-)$ system in its rest frame; θ is the polar angle of the $\rho_1^0 = \pi_1^+ + \pi_2^-$ with respect to the $\gamma\gamma$ axis (the corresponding azimuthal angle can be set to zero, without loss of generality); $\Omega_1 = (\theta_1, \phi_1)$ describes the direction of π_1^+ in the helicity coordinate system[10] defined in the rest frame of the ρ_1^0; and, likewise, $\Omega_2 = (\theta_2, \phi_2)$ describes the direction of π_3^+ in the rest frame of the

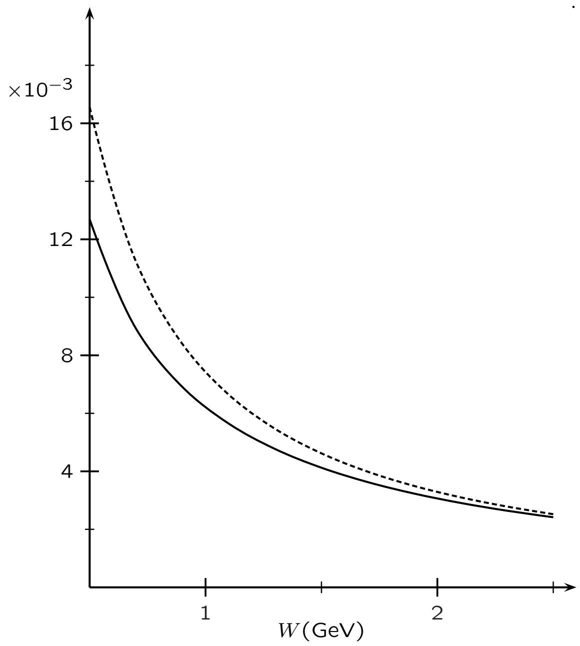

Figure 2: Same as Fig.1, except that the rapidity is $y = \pm 1$.

ρ_2^0. The differential cross section for the process (59) can be written

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \propto \frac{1}{W^2}\, I(\tau)\, g(\tau) \tag{61}$$

where τ represents a set of 7 variables as specified in (60). We have fixed the flux factor by $\mathcal{F} = p_\gamma W = W^2/2$, assuming that the cross section formula above refers to the case in which the photons are massless, i.e. $p_\gamma = W/2$.

The amplitudes discussed in Section 3 can now be given

$$\begin{aligned} A(J^{PC}\Lambda;\tau) = &\left(\frac{2J+1}{4\pi}\right)^{1/2} \left(\frac{2s_1+1}{4\pi}\right)^{1/2} \left(\frac{2s_2+1}{4\pi}\right)^{1/2} \\ &\times \sum_{\mu_1\mu_2} G^a_{\mu_1\mu_2}\, d^J_{\Lambda,\mu_1-\mu_2}(\theta)\, D^{s_1\,*}_{\mu_1,0}(\phi_1,\theta_1,0)\, D^{s_2\,*}_{\mu_2,0}(\phi_2,\theta_2,0) \end{aligned} \tag{62}$$

where $s_1 = s_2 = 1$ stands for the ρ spin, and μ_1 and μ_2 are the helicities of the ρ_1^0 and ρ_2^0, respectively. $G^a_{\mu_1\mu_2}$ is the helicity-coupling amplitude for the $\rho^0\rho^0$ system, the equivalent of the F^a's for $\gamma\gamma$ states considered in Section 3.

The helicity-coupling amplitudes G^a's can be expressed

$$\begin{aligned} G^a_{\mu_1\mu_2} = &(k_1)^{s_1}\,\Delta(w_1)\,(k_2)^{s_2}\,\Delta(w_2) \\ &\times \sum_{LS} \left(\frac{2L+1}{2J+1}\right)^{1/2} p_\rho^L\, R^a_{LS}\,(L0\,S\delta|J\delta)\,(s_1\mu_1\,s_2\,{-\mu_2}|S\delta) \end{aligned} \tag{63}$$

where $\delta = \mu_1 - \mu_2$. $\Delta(w)$ is a relativistic Breit-Wigner form for the ρ; p_ρ^L, $(k_1)^{s_1}$ and $(k_2)^{s_2}$ are the appropriate angular-momentum barrier factors, which are replaced in practice by the Blatt-Weisskopf form factors[17]; the usual Clebsch-Gordan coefficients are denoted $(j_1 m_1 j_2 m_2 | jm)$; and R^a_{LS} is the parameter to be determined from data. For each given $a = J^{PC}$, there exists a set of allowed $\{L, S\}$, i.e. $\eta_a = (-)^L$, $|L - S| \leq J \leq L + S$ and $\{S = 0, 1, 2\}$, with a contraint that $L + S =$ even, which comes from the presence of identical ρ^0's in the intermediate state. The photon helicities $\{\lambda_1\, \lambda_2\}$ have been suppressed from the amplitude A, $G^a_{\mu_1 \mu_2}$ and R^a_{LS}, respectively, for ease of notation.

We note that the formula (63) is non-relativistic. It lacks the functional dependence on the Lorentz factors, $\gamma_1 = E_1/w_1$ and $\gamma_2 = E_2/w_2$, which must be present, since the intermediate states ρ_1 and ρ_2 have finite momenta in the two-photon center-of-mass system. Essentially, the decay amplitudes in general must depend not only on the momenta p_ρ, k_1 and k_2 but also on the energies E_1 and E_2 defined in the overall rest frame. The reader may consult Ref.[18] for the general prescription of such a relativistic formalism.

Finally, a comment is in order on the shape of the W spectrum. For the purpose, consider the integrated cross section

$$\sigma(W) \propto \frac{1}{W^2} \int I(\tau)\, g(\tau)\, \mathrm{d}\tau \tag{64}$$

It is clear that $\sigma(W)$ as a function of W has no physical meaning, i.e. the formalism given here cannot say anything about the shape of the W spectrum. It is given instead by the convolution of the two-photon physical process, which we wish to explore, and the experimental acceptance.

References

[1] S. U. Chung, D. P. Weygand and H. J. Willutzki, 'Double-Pomeron and Two-Photon Processes at RHIC,' BNL-QGS-02-91; S. U. Chung and H. J. Willutzki, 'Hadron Spectroscopy and B Physics at RHIC,' BNL preprint (1990); S. U. Chung, 'Reflectivity Eigenstates for Central Production,' BNL-QGS-94-23.

[2] N. Isgur and J. Paton, Phys. Rev. D **31** (1985) 2910.

[3] N. Isgur, R. Kokoski and J. Paton, Phys. Rev. Lett. **54** (1985) 869; F. E. Close and P. R. Page, Nucl. Phys. B **443** (1995) 233; T. Barnes, F. E. Close and E. S. Swanson, Phys. Rev. D **52** (1995) 5242.

[4] S. U. Chung, 'Quantum Numbers for Hybrid mesons in the Flux-tube Model(Version III),' CERN-QGS-00-1201.

[5] D. R. Thompson, *et al.*, Phys. Rev. Lett. **79** (1997) 1630.

[6] A. Abele *et al.*, Phys. Lett. **B423** (1998) 175.

[7] A. Abele *et al.*, Phys. Lett. **B446** (1999) 349.

[8] G. Adams *et al.*, Phys. Rev. Lett. **81**(1998) 5760.

[9] V. Dorofeev, *Proceedings of the Workshop on Hadron Spectroscopy, Frascati, Italy (1999),* edited by T. Bressani, A. Feliciello and a. Filippi (Frascati Physics Series, Vol XV), p. 3.

[10] S. U. Chung, 'Spin Formalisms,' CERN preprint 71-8 (1971).

[11] M. Jacob and G. C. Wick, Ann. Phys. (USA) **7**, 404 (1959).

[12] F. Close *et al.*, Phys. Rev. D **57** (1997) 5653; P. R. Page, Acta Physica Polonica B **29** (1998) 3387.

[13] P. R. Page, private communication.

[14] A. Anastassove *et al.* Phys. Rev. Lett. **84** (2000) 1393.

[15] S. U. Chung, 'Double-Pomeron and Two-Photon Processes at RHIC,' BNL preprint BNL-QGS-02-91.

[16] See, for example, a note written by R. N. Cahn, in 'Review of Particle Physics,' Particle Data Group, Eur. Phys. J. **C 15**, 1 (2000). See also V. M. Budnev *et al.*, Phys. Reports **15C**, 181 (1975).

[17] S. U. Chung, Phys. Rev. D **48**, 1225 (1993); F. von Hippel and C. Quigg, Phys. Rev. **5**, 624 (1972).

[18] S. U. Chung, Phys. Rev. D **57**, 431 (1998).